ON DELIVERY POLICIES FOR A TRUCK-AND-DRONE TANDEM IN DISASTER RELIEF

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ABSTRACT

This paper introduces the Steiner traveling salesman problem with a truck and a drone with incomplete information (STSP-DI). STSP-DI is motivated by the deliveries of emergency supplies under unknown road conditions in the immediate aftermath of a disastrous event. Some roads are impassable for the truck, but the drone can fly over the road damages. The necessity to provide medical assistance and meet the basic needs of those affected, forces deliveries to begin as soon as possible, even though the condition of some roads is still unknown and must be scouted by the vehicles 'on-the-fly'. The relief group must schedule deliveries anticipating possible unplanned truck detours, enforce (planned) detours for early testing of key road segments, and consider the dynamic nature of road condition information.

In this paper, we perform a competitive analysis of a widely used delivery policy for the STSP-DI in practice - the online re-optimization policy (Reopt) - and compare it to several alternative delivery strategies that are easy to implement in disaster response. Most of the calculated competitive ratios are sharp. Competitive analysis examines the worst-case performance of the strategies and is particularly important in the context of disaster relief, where the worst-case outcomes must be avoided.

Our analysis shows that Reopt is dominated by alternative delivery policies in terms of the competitive ratio even at a medium level of damage on the road. It also underscores the importance of surveillance detours performed by the drone, even if the surveillance delays the start of the deliveries.

Keywords Scheduling, Traveling Salesman Problem with a Drone, Disaster Relief, Competitive Analysis

1 Introduction

Driven by climate change, the destructive force of weather-related disasters has intensified, and the frequency of those events has increased by a factor of five over the past 50 years (WMO, 2021). During or in the aftermath of a natural disaster, timely response is absolutely essential. A response includes distribution of emergency supplies, such as medication, water, toolkits, and communication devices, to the impacted individuals. However, road infrastructure may be severely damaged and some roads may become impassable for vehicles. Aggravating this situation, dense cloud coverage, smoke, and vegetation at the roadsides may make satellite pictures of the terrain uninformative, and the state of many roads may remain unknown for a prolonged period of time (American Red Cross and Measure, 2015; Iqbal et al., 2007). As a result, sometimes even available supplies cannot be distributed in time, leading to severe, potentially life-threatening shortages (Iqbal et al., 2007). Emergency deliveries may be significantly improved by utilizing novel robotic technology – *unmanned aerial vehicles* (*UAVs*), or *drones* (see Figures 1,2) (American Red Cross and Measure, 2015; Ashley, 2017; Hermant, 2016), see the survey (Otto et al., 2018a).

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Figure 1: A drone transports a lunchbox to an elderly person, who resides in a remote village cut off by road damage in the municipality Penela, Portugal

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Figure 2: A drone delivers prescription medication to a remote hospital in Texas

In this article, we examine the supply delivery by a truck and a drone as part of the immediate response phase. This means that the delivery vehicles can depart immediately, tolerating the uncertainty on the traversability of the truck's scheduled route. What comes out is an online optimization problem, since the condition of single road segments is revealed dynamically during the mission as the truck or the drone approach the impassable segment. This requires from the developed delivery policies a real-time adjustment of routing decisions at each edge blockage. We formulate the delivery problem as the *Steiner Traveling Salesman Problem with a truck and a drone with incomplete information* (*STSP-DI*). We call it Steiner, because some nodes are not required to be visited and any edge or node in the street-network graph can be visited multiple times (cf. Álvarez Miranda and Sinnl, 2019). To provide the first impression on the STSP-DI, Figure 3 illustrates an instance and a possible delivery plan for the *complete-information counterpart* of the STSP-DI, i.e., for the problem in which the status of all the edges is known. The STSP-DI resembles the Traveling Salesman Problem with a drone (TSP-D) (Agatz et al., 2018; Murray and Chu, 2015), because both the truck and the drone can perform deliveries, the drone can carry one package at a time and the drone has to meet the truck periodically in one of the admissible rendezvous nodes to pick up packages. However, there are two main differences between our problem and the TSP-D. Firstly, we allow the drone to take off and land on the truck at nodes that are not required to be visited. Secondly, the information on the edge status is incomplete.

We examine several important routing policies for the STSP-DI and provide their *competitive ratios*. Consider an online minimization problem, i.e., a problem with incomplete information, where some of this information can be revealed in the future. Let A be an online approximation algorithm. The *competitive ratio* $\sigma(A)$ is the worst-case ratio of the online algorithm's cost to the cost of an optimal offline algorithm, where all data are known a priori (Jaillet and Wagner, 2008a). If $A(I)$ is the objective value of algorithm A on problem instance I, I^* is the corresponding instance with complete information and $OPT(I^*) > 0$ is the optimal objective value of this instance, then

$$\sigma(A) = \sup_I \frac{A(I)}{OPT(I^*)}.$$

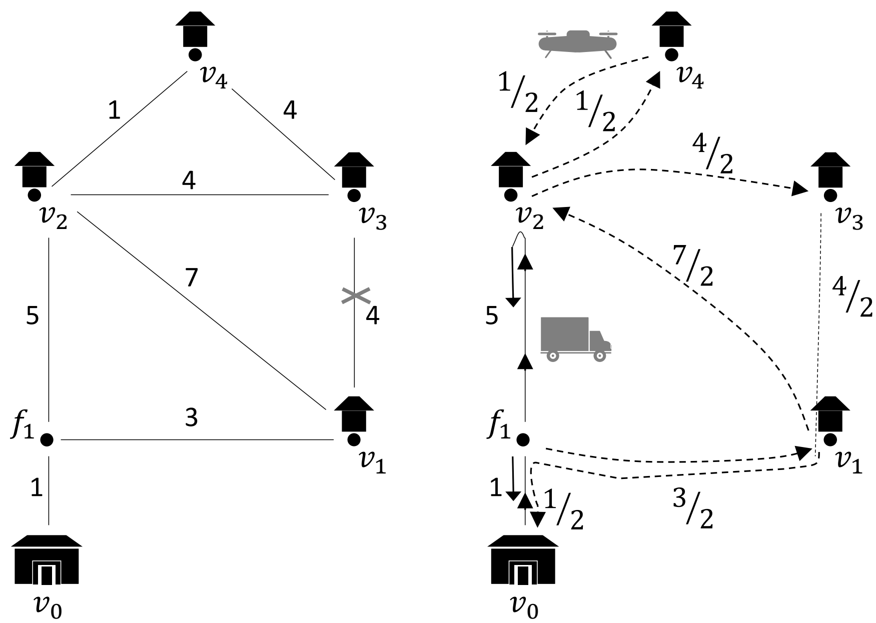

Figure 3: An instance of the *complete-information counterpart* of the STSP-DI with one damaged edge (v_1, v_3) (left) and an optimal solution for this instance (right)
Left figure: Edge labels show truck travel times. The drone is $\alpha = 2$ times faster than the truck. Nodes $v_i \in V$ with $i > 0$ are required to be visited at least once, node f_1 is an optional (Steiner) node, and v_0 denotes the depot.
Right figure: The truck walk is depicted with solid lines, the drone sorties with dashed lines. Edge labels depict travel times of the corresponding vehicle: of the truck for solid edges and of the drone for dashed edges. The optimal solution to the right consists of one travel leg (v_0, f_1) and three operations $((f_1, v_2)(f_1, v_1, v_2))$, $((v_2)(v_2, v_4, v_2))$ and $((v_2, f_1, v_0)(v_2, v_3, v_1, f_1, v_0))$ and has length 13. The corresponding walks of the truck and the drone are $\pi^t = (v_0, f_1, v_2, f_1, v_0)$ and $\pi^d = (v_0, f_1, v_1, v_2, v_4, v_2, v_3, v_1, f_1, v_0)$.

The exact value of $\sigma(A)$ is important for decision making, since the competitive ratio offers crucial insights for disaster relief management. These insights, for instance, cannot be gleaned by running simulations. First of all, unlike simulations, it provides a performance guarantee for the delivery policy in any possible scenario. Further, if delivery times in some disaster relief operation turned out to be very long, the competitive ratio indicates how much of the delivery time is attributed to the severity of the disaster and how much can be potentially shortened by improving delivery policies. Complete information in the STSP-DI implies having a perfect damage detection technology that provides all the information on the road conditions at once. In this respect, $\sigma(A)$ also offers, indirectly, some insight on the importance of innovation in the damage detection technologies.

Observe that the competitive ratio resembles the concept of ρ-approximation in the offline optimization literature (cf. Borodin and El-Yaniv, 2005), which compares the algorithm's result with the optimal objective value of the same instance. In a certain sense, $\sigma(A)$ can be seen as an extension of ρ-approximation for the case of incomplete information.

Our contribution is as follows:

- We calculate and prove competitive ratios of several important routing policies for the truck and the drone in the STSP-DI. To the best of our knowledge, we are the first ones to perform the competitive ratio analysis in the context of truck-and-drone routing.
- The conducted competitive analysis is parametric, meaning that the competitive ratios are provided granularly, for each possible constellation of three characteristic parameters.
- Based on our analysis of the competitive ratios, we formulate several nontrivial managerial insights. For instance,
  - our analysis underscores the role of information collection and we show that in many cases the drone should take a detour to examine the status of some pivotal road segments in advance.
  - our analysis also reveals that a straightforward online re-optimization policy, when the current delivery plan is re-optimized as soon as a (relevant) damaged edge has been encountered, has a bad competitive ratio and is outperformed by several alternative policies by a large margin.

We proceed with the literature review in Section 2 and a problem description of STSP-DI in Section 3 followed by a summary of our competitive ratio results for three important policies in Section 4, which is a summary of the main results of this paper. This section can be understood without going into the technical details of the proofs presented in Section 5. We conclude with a discussion and an outlook in Section 6.

2 Literature review

Studies on humanitarian logistics, including routing of supply vehicles in the aftermath of a disaster, have attracted a considerable amount of attention among many researchers and practitioners in recent years, see the reviews of Anaya-Arenas et al. (2014); Özdamar and Ertem (2015); Farahani et al. (2020) and Kundu et al. (2022). Driven by the specificity of the disaster relief applications, two major differences emerge compared to the 'classical' routing problem formulations. First of all, this is the importance of time, which usually enters the models as part of an objective function, such as minimize maximal or average arrival time (cf. Campbell et al., 2008; Huang et al., 2013; Sabouhi et al., 2018). Second, this is the deployment of heterogeneous vehicles, which differ in their capacity, cost, and the purpose of use.

Much research effort assumes *complete information* on the road damages and investigates the routing of the repair crew to restore the network connectivity in minimum time. In particular, Duque et al. (2016) propose an exact method and a greedy-randomized constructive heuristic for the network repair crew scheduling and routing problem, with the objective of minimizing the weighted accessibility time of each recipient of relief. Moreno et al. (2020) extend the formulation with novel models and valid inequalities. The models investigated by Yan and Shih (2009) and Shin et al. (2019) integrate both, deterministic emergency repair and relief distribution. Yan and Shih (2009) build time-space networks for a bi-objective model, minimizing the length of time required for both tasks, while Shin et al. (2019) use an ant colony algorithm to minimize the time of the relief distribution after the roads turn accessible.

Much less attention is paid to a more realistic setting of *incomplete information* on the state of the infrastructure. The available research focuses on two-stage algorithms for relief distribution performed by *(homogeneous) trucks*, in which uncertainty impacts only certain long-term decisions, such as the location of depots or fleet sizing, and the information on the road condition is revealed in the second stage, when the delivery routes of the vehicles are planned (e.g., Ahmadi et al., 2015; Moreno et al., 2018; Rath et al., 2016; Salmerón, 2010). Only a few articles investigate cases, when relief distribution is performed by trucks *and drones* (Golabi et al., 2017; Chowdhury et al., 2017). The authors determine locations of aid distribution centers, from which trucks and drones will deliver aid to the impacted individuals, with the objective to minimize distribution time or cost. Chowdhury et al. (2017) utilizes the methods of continuous approximation, whereas Golabi et al. (2017) optimizes over a set of likely scenarios in which specific areas turn inaccessible. Several studies investigate a pure monitoring or assessment of damage by off-road vehicles such as drones or motorcycles without considering the *following* relief *distribution* (e.g., Oruc and Kara, 2018; Zhen et al., 2019; Zhang et al., 2021; Reyes-Rubiano et al., 2021, 2022). To the best of our knowledge, only Macias et al. (2020) and Farzaneh et al. (2023) consider simultaneous relief distribution and network damage assessment *by drones*. However, in both articles, only trucks perform deliveries of emergency supplies; in these problem formulations, the advantage of the drone's capability to fly over road debris is ignored. In Macias et al. (2020), the damaged edges remain passable for the trucks, just the truck traversal times increase, which is different from our model. Macias et al. (2020) consider the normal distribution for the truck traversal times on damaged edges and propose a greedy-based heuristic and a genetic algorithm for the formulated problem. Farzaneh et al. (2023) consider an integrative heuristic planning framework, which includes the road damage assessment by drones, road recovery and relief distribution by trucks. Given the scope of this planning problem, a number of restrictive assumptions have to be necessarily imposed. For instance, the drone routing for damage assessment *does not anticipate* the impact of the gained information on the relief distribution; the objective is to scout the impacted region completely at minimum cost. Furthermore, the relief distribution framework is very different: trucks attend only one customer (demand node) per trip in a direct way there and back from the depot. In Farzaneh et al. (2023), trucks can only use roads, which are known to be intact, and disrupted roads can be repaired after a certain amount of time by recovery teams. To the best of our knowledge, no studies on truck-drone routing policies for simultaneous network damage assessment and deliveries in case of impassable road blockages have been performed so far. Moreover, the competitive ratio analysis for truck-and-drone missions in disaster relief that we present is new.

A separate thread of the literature studies the quality of *online algorithms* in routing applications with incomplete information. The most common state-of-the-art method remains the competitive ratio analysis. Ausiello et al. (2001) were the first ones to consider an online variant of the TSP, where the requests (i.e, the nodes to be visited) appear dynamically over time, and they propose a polynomial online algorithm with the best-possible competitive ratio. Afterwards, competitive ratios have been examined for a number of variants of this TSP with online requests, such as with precedence, time and capacity constraints, with the flexibility to drop some nodes, with several salesmen or for variants of the vehicle routing problem (Jaillet and Lu, 2011, 2014; Jaillet and Wagner, 2008a,b; Larsen et al., 2007; Wen et al., 2012, 2015). From the analytical point of view, the aforementioned TSP problems with node uncertainties

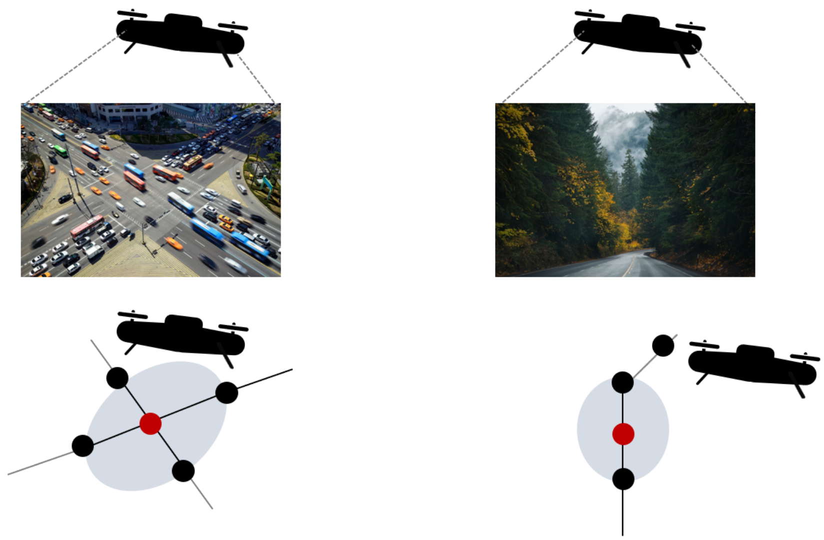

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Figure 4: Illustrative example for modeling the visibility of the roads
The truck and the drone only see the travel conditions of an edge after reaching one of its defining nodes. The center node in the gray area at the bottom of the figure marks the current position of the vehicle, the remaining nodes are left dark.

deviate significantly from the setting studied in this paper. Liao and Huang (2014) were the first to study an online TSP with uncertainty about edge traversability, which is more similar to our situation. They called the problem Covering Canadian Traveler Problem (CCTP), in reference to its previous shortest-path version – the Canadian Traveler Problem –, and propose a touring policy with an attractive competitive ratio for one traveler who dynamically encounters blocked edges on her pass, in case the graph remains connected. Zhang et al. (2015) add Steiner nodes to this formulation and propose an exponential-time best possible online algorithm in terms of the competitive ratio and a polynomial-time well-performing algorithm. Further extensions and variations of the CCTP and the Steiner Traveling Salesman Problem with online edge blockages have been studied in recent years, for example, with multiple agents (Akbari and Shiri, 2022; Liu et al., 2021; Shiri et al., 2020; Zhang et al., 2022), the flexibility to drop some nodes (Büttner and Krumke, 2016), advanced information on blocked edges (Zhang et al., 2016), or with the minimum latency objective (Akbari and Shiri, 2022, 2021; Zhang et al., 2019). The STSP-DI is different from the studied problems as it involves a delivery drone with distinctive features, such as limited payload capacity coupled with the ability to fly over the blocked edges. Additionally, nontrivial *synchronization* of the delivery tours of the truck and the drone invalidate the results of competitive analysis for the truck-only case.

To the best of our knowledge, we are the first ones to perform the competitive ratio analysis for truck-and-drone routing in disaster relief. Moreover, it is the first study on the online delivery policies for the truck and the drone in case of road blockages and incomplete information on the status of the roads. In the preliminary work of the authors, several heuristic algorithms are presented for the problem variant of the STSP-DI with given probabilities of arc damages (Otto et al., 2018b).

3 The Steiner Traveling Salesman Problem with a truck and a drone under incomplete information

The Steiner Traveling Salesman Problem with a truck and a drone under incomplete information (STSP-DI) is to find a tour for a truck and a drone with a minimal duration (makespan) that starts at the depot, delivers packages to all the addresses and ends at the depot. In this setting, information on edge damages is initially incomplete.

We describe a street network as a simple undirected graph $G = \left(L, E, c^t, c^d\right)$ where L is the set of nodes, E defines the set of edges and c^t and c^d are non-negative edge weights. Set L consists of three groups of heterogenous nodes (see Table 1).

The subset of nodes $V \subseteq L$ refers to delivery addresses. These nodes are required to be visited at least once. The truck and the drone have to deliver a set of packages, each of which is associated with a delivery address $v_i \in V$. Having widely available drone technologies in mind, we assume that the drone can only carry one package at a time and that it picks up packages from the truck.

Set $D \subseteq L$ denotes locations where the drone can safely take off from the truck or land on the truck, such as parking lots. Set D includes delivery addresses $v_i \in V$, the depot v_0, and possibly some Steiner nodes f_i. Although we allow the drone to launch from and land on the truck at the nodes of both types v_i and f_i, we only need to deliver to v_i, which is the main reason why we use different letters for these two types of addresses.

The set of nodes L may contain further Steiner nodes $l_i \in L \setminus D$ to reflect specialties of information acquisition. The motivation for these additional nodes l_i is as follows. The truck and the drone spot damaged road segments as they approach them. Roads may have very different visibility. A curvy road in a dense forest or in a dense urban terrain has very poor visibility, whereas straight road segments in flat terrain may be in clear view for many kilometers. Therefore, we suggest a discretization approach where, depending on visibility, additional *lookout* nodes $l_i \in L \setminus D$ are defined on each path (road). Drone or truck can see the travel conditions of edge $(i, j) \in E$ when they reach one of its nodes $i, j \in L$, see Figure 4. This widespread information acquisition modeling approach is used, for instance, in the Canadian traveler problem (Aksakalli et al., 2016; Papadimitriou and Yannakakis, 1991). Defining lookout nodes at varying distances allows one to flexibly model various terrains, e.g., a dense discretization can be used for low visibility. Table 1 sums up different types of nodes in L.

Table 1: Types of nodes in the STSP-DI

Set of nodes L and its subsets	Function
L	The set of nodes in the STSP-DI. At each node, the truck and the drone receive information on the status of the adjacent edges.
$D \subseteq L$	Safe locations for the truck and the drone to meet; the set includes delivery addresses and the depot v_0.
$V \subseteq D$	Delivery addresses, which must be visited at least once.

Set $E \subseteq \{(i, j)|i, j \in L\}$ is a set of undirected edges, each one describing road segments between nodes $i, j \in L$, some subset $\mathcal{E} \subseteq E$ of which is damaged. Note that the truck cannot traverse a damaged edge. We also assume that the drone can't fly shortcuts, but can traverse damaged edges that wouldn't allow the truck to continue. Initially, the status of edges (damaged or intact) is unknown, except for those edges adjacent to the depot v_0. Edge weights $c^t(i, j) \geq 0$ and $c^d(i, j) \geq 0$ denote the travel times on edges $(i, j) \in E$ for the truck and the drone, respectively. The drone is α times faster than the truck so that $c^t(i, j) = \alpha \cdot c^d(i, j), \alpha > 0$. Observe that $c^d(i, i) = c^t(i, i) := 0$ for all $i \in L$. Delivery times at any $v_i \in V$ are set to be zero.

Consider the illustrative example of Figure 3, which contains one damaged edge (v_1, v_3). In the case of *complete information*, the damage of edge (v_1, v_3) is known; in an optimal solution, the drone travels on the truck that departs from v_0 to f_1, then it launches to perform delivery to v_1 and returns back to the truck at node v_2, thereafter, it performs a loop-sortie at v_2 to deliver to v_4, and finally it launches at v_2 to deliver to v_3 and returns along the shortest route to the depot v_0. We emphasize that f_1 denotes a safe location (Steiner point), whereas v_1, v_2, v_3, and v_4 denote delivery addresses. The truck and the drone may need to wait for each other in order for the drone to pick up or deliver a package. So that it is convenient to think about a delivery solution as a sequence of *operations* and *travel legs*. Each operation is associated with one delivery performed by the drone and consists of two subsequences of nodes: the truck walk and the drone walk that start at the same launching node and end at the same return node. In a travel leg, the drone travels on the roof of the truck. The solution in Figure 3 consists of one travel leg (v_0, f_1) and three operations $((f_1, v_2)(f_1, v_1, v_2))$, $((v_2)(v_2, v_4, v_2))$, and $((v_2, f_1, v_0)(v_2, v_3, v_1, f_1, v_0))$, which result in the truck walk $\pi^t = (v_0, f_1, v_2, f_1, v_0)$ and the drone walk $\pi^d = (v_0, f_1, v_1, v_2, v_4, v_2, v_3, v_1, f_1, v_0)$.

In a given solution s, the duration $\tau_o(s)$ of operation $o(s)$ is the maximum of the truck travel time $\tau_o^t(s)$ and the drone travel time $\tau_o^d(s)$ in this operation: $\tau_o(s) = \max\{\tau_o^t(s), \tau_o^d(s)\}$. Here and in the following, we simplify notation and skip the reference to solution s, whenever this solution is clear from the context. For example, we write τ_o instead of

$\tau_o(s)$. On a travel leg, the drone travels on the roof of the truck, so that the duration of the travel leg depends on the truck travel time only.

The duration or *makespan* $T(s)$ of solution s is the time from when the truck and/or the drone jointly leave the depot for the first time until they return to the depot after having completed the required deliveries. It equals the sum of durations of the travel legs and the operations. The duration of the travel leg in the solution of Figure 3 is 1 and the durations of three operations are 5, 1, and 6, so that the makespan of this solution equals 13. It is an optimal solution for $\alpha = 2$ and the complete-information instance as shown in Figure 3.

We denote an instance with the complete information on edge damages as I^*, the makespan of the optimal solution for instance I^* as $T^* = OPT(I^*)$, and an optimal solution of I^* as s^*.

We assume that the truck has a speed of 1 and that the drone has a speed of α. We call a set of addresses that share the same location an *agglomeration*. Formally, if some $v_i, v_j \in V$ belong to the same agglomeration, then $c^t(v_i, v_j) = c^d(v_i, v_j) = 0$, the status of edge (v_i, v_j) is known and it is not damaged. For example, in Figure 8, nodes v_1 to v_n form an agglomeration.

We make the following assumptions in our problem formulation:

- *Immediate information transmission*. As soon as either truck or drone has an update on the status of a road segment, the information immediately reaches the other vehicle as well.
- *Unlimited memory*. No information about road segments gets lost.
- *No repairs and no further damages*. The set $\mathcal{E}$ of damaged edges does not change.
- *Identical packages*. All packages are treated to be identical and the truck carries a sufficient number of packages on board.
- *Infinite energy and communication radius*. There is no limit on the length of the drone sortie in an operation.
- *The same fields of view of the truck and the drone*. For simplicity of exposition, we assume the set of lookout nodes L to be the same for the truck and the drone, i.e., the fields of view of the truck and the drone are identical. Note, however, that, for the most part, the conclusions reached in this article are still valid if this assumption is relaxed.

4 Overview of competitive ratios of several important planning policies

In this article, we analyze the competitive ratio for the in practice widely used optimistic online re-optimization policy *(Reopt)* and two alternative more conservative policies – the conservative delivery policy *(CD)* and the surveillance-first policy *(SF)*.

Optimistic online re-optimization. In Reopt, the planner assumes all the edges with unknown status are intact. Therefore, the truck and the drone start performing deliveries based on an optimal solution of the resulting STSP-DI instance with no edge damages. In the course of their walks, the truck or the drone might discover damaged edges that affect the currently planned truck route. Each time such a damage is discovered by either of the vehicles, the delivery walks of both vehicles are re-optimized given their current positions and assuming all the edges with yet unknown status are intact (see Section 5.1 for details). Reopt is considered attractive by practitioners, because deliveries start immediately and are performed in the shortest possible time, if there are no damaged edges on the truck walk.

Conservative delivery. In CD, deliveries are performed only by the drone, which collects a package at the depot v_0 each time and makes a return flight to a customer along the shortest trajectory. This policy is the best possible algorithm if all the edges are damaged.

Surveillance first. In SF, the initial truck and drone delivery walks are constructed under the assumption that all edges with unknown status are intact. But before the truck departs from the depot, there is a surveillance phase when the drone clarifies the status of all the edges of the planned truck walk starting from the depot. Each time the drone discovers that an edge of the truck walk is damaged, a new delivery tour for the truck and the drone is calculated assuming all the edges with yet unknown status are intact. Both vehicles do not start deliveries before the drone has returned to the depot and has confirmed that all the edges in the truck walk of the currently planned delivery solution are intact (see Section 5.3 for details). In disaster relief, the decision to pause deliveries until more information on road status is collected is always difficult because each minute counts. Therefore, the surveillance-first policy seems overly conservative for many practitioners and its benefits are not intuitive.

In our analysis, we parametrize the competitive ratio by the following instance parameters, i.e., we provide the value for the competitive ratio for each combination of the parameter values:

- The number of damaged edges $k \in \mathbb{N} \cup \{0\}$.
- The ratio α between the travel time of the truck and the drone.
- The number of required deliveries $|V| \in \mathbb{N}$.

Table 2 summarizes the competitive ratios of the described delivery policies, proofs of which can be found in the following sections. Table 2 reveals that apart from a straightforward case of $|V| = 1$ and $\alpha > 1$, when the drone performs the only delivery and all the presented policies find $OPT(I^*)$, the competitive ratios may differ a lot. A numerical example with a typical ratio of the drone and the truck speed $\alpha = 2$ (cf. (Poikonen and Golden, 2020; Stolaroff et al., 2018)) illustrates that the widespread Reopt may be significantly outperformed by more conservative strategies CD and SF in realistic settings.

Table 2: Parametric competitive ratios of some delivery policies

Optimistic online re-optimization	Conservative delivery	Surveillance first
$\begin{cases} 1 & \text{if } \lvert V\rvert = 1, \alpha > 1, \\ \geq 2^k & \text{otherwise.} \end{cases}$	$\begin{cases} 1 + \frac{\lvert V\rvert-1}{\alpha} & \text{if } \alpha \geq 1, \\ \frac{\lvert V\rvert}{\alpha} & \text{if } \alpha < 1. \end{cases}$	$\begin{cases} 1 & \text{if } \lvert V\rvert = 1, \alpha > 1, \\ 1 + \frac{k+1}{\alpha} & \text{otherwise.} \end{cases}$
Illustration for $k = 5$, $\lvert V\rvert = 30$, and $\alpha = 2$		
≥ 32	15.5	4

Next, we summarize managerial insights based on our analysis of the competitive ratios.

Reopt is optimal whenever the truck encounters no damages in its walk ($k = 0$). But it is a costly gamble. If several damaged edges are encountered, Reopt may end up generating extremely poor solutions: myopic truck drivers may select very risky routes based on the slightest promise of shorter delivery times and may have difficulties in returning to the depot. In particular, the worst-case for Reopt tends to occur when there are multiple consecutive pathways between a pair of delivery addresses that are discovered as impassable, which eventually causes a long backtrack. Since damaged edges are often clustered in highly impacted regions, such blocking of multiple alternative pathways can reasonably occur. For example, flooding or storm surge could cause clusters of road segments to become impassable leading to massive backtracking, especially if an island or coastal area is impacted. If the drone is faster than the truck ($\alpha > 1$), the more conservative policy SF dominates Reopt already for $k \geq 2$ in terms of the competitive ratio.

The actual behavior of the drivers seem to mimic the Reopt policy. The following typical behaviour – risk seeking in the domain of losses – is described in psychology (Kahneman and Tversky, 2000): If the driver has a choice to take a surely undamaged path or to take a shorter risky path, which may require a long detour (=losses) with some small probability, he/she prefers, at least occasionally the latter even if the expected loss outweighs the expected benefit. Indeed, an assessment report of disaster relief planning and execution recounts that dispatched delivery trucks sometimes get lost and do not return to the depot for a surprisingly long time (Iqbal et al., 2007).

As expected, CD may become very attractive if the drone is fast and the number of addresses is rather small. For example, if $\alpha > 1$, it dominates SF if $|V| < (k + 2)$.

SF performs surprisingly well if measured by the competitive ratio. It may outperform Reopt even if the drone is slow ($\alpha < 1$), the advantage grows quickly with each damaged edge in the truck walk. In unfortunate instances, as depicted in Figure 10, it does matter whether the truck can discard some impassable paths early on, because the length of the detour is quite large. To sum up, an optimal delivery policy should include surveillance in one form or another as soon as the truck should deliver to some of the addresses.

Observe that competitive analysis makes no statement about the average performance of the algorithms. Even if the worst-case performance of some policy is bad, it may perform quite well on average. Yet, in disaster relief, worst outcomes are to be avoided. Therefore, the knowledge of the competitive ratios should complement the simulation results as an assessment tool for the selection of a suitable policy.

5 Proof of competitive ratios

In this section, we provide the proofs for the competitive ratios, which were stated in Section 4.

In the following, we refer to the truck and the drone solving instance I as *a policy solution* (PS), and to those optimally solving instance I^* as *a complete information optimal solution* (CIOS). To streamline our exposition, we make the

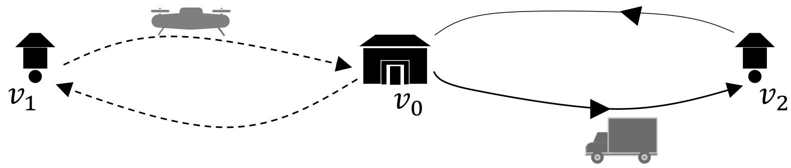


Figure 5: Illustration for Lemma 1: An instance with $V = \{v_1, v_2\}$
The figure visualizes only a portion of the graph, the k damaged edges are located somewhere in the not-visualized part of the graph. Distances are selected such that the drone travels the left loop, and the truck travels the right loop; both vehicles return at the same time T^* to the depot in a complete information optimal solution (CIOS).

following two assumptions. First, we have to decide how to proceed with the cases of ties. We assume that in case of ties, a policy selects the *worst possible* decision for the examined instance. For example, if $|V| = 1$ and the truck and drone are equally fast with $\alpha = 1$, we assume that the Reopt-planner sends the truck to perform this only delivery. An alternative tie-breaker would not change the general direction of our managerial insights and conclusions. Nevertheless, we carefully mark the cases, where this technical assumption is used as well as highlight its consequences. Secondly, if we have to construct an unfavorable instance with a large number of delivery addresses $|V|$, we usually place most of them in one location as an agglomeration. Although this serves as a simplification, agglomerations are quite common in disaster relief. For instance, a village in a vast, but sparsely populated area or a dormitory suburb of a city can be modeled as agglomerations. We use $[a, b] := \{a, \dots, b\}$ to denote the set of integers between a and b, $a, b \in \mathbb{Z}$. We start with Lemma 1, which we use throughout the section.

Lemma 1. *For any $|V| \geq 2$, $\alpha > 0$ and $k \in \mathbb{N} \cup \{0\}$ we can always construct an instance where the set of locations V is divided into two subsets V_1 and V_2 and the damaged arcs are distributed such, that in a complete information optimal solution (CIOS) the drone delivers to all $v \in V_1$, the truck to all $v \in V_2$, and both finish at the same time T^*.*

Proof. Figure 5 provides an illustration. There are two loops on opposite sides of the depot v_0, consisting of two or more edges each, none of them is damaged. Address v_1 is located in the middle of the left loop, and the remaining addresses $V \setminus \{v_1\}$ are placed in the middle of the right loop of total length T_r. Recall that the speed of the truck is normalized to 1 and the speed of the drone equals α. The length of the left loop is $T_l = \alpha \cdot T_r$. The rest of the graph is not visualized and can be chosen arbitrarily provided that the left and the right loops remain shortest paths from the depot v_0 to v_1 and from the depot v_0 to all addresses $V \setminus \{v_1\}$ in I^*, respectively. All k damaged edges can be located arbitrarily in that hidden portion of the graph. Obviously, the vehicles travel $\frac{T_l}{\alpha} = T_r$ and T_r time, respectively, and return at the same time to the depot. □

Our proofs have the following similar structure:

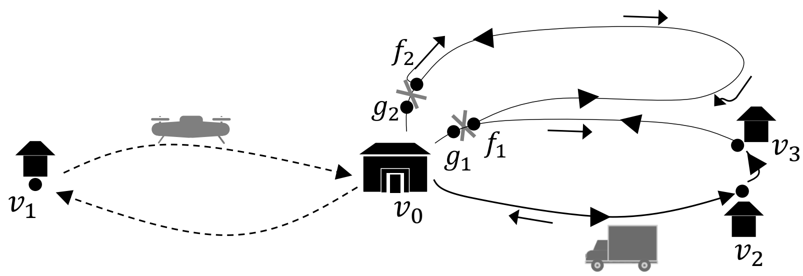
- we first find an unfavorable instance I' such that $\frac{A(I')}{OPT(I'^*)} = w$ for some $w \in \mathbb{R}^+$.
- then, we prove that there exists no instance I with $\frac{A(I)}{OPT(I^*)} > w$.

5.1 Analysis of the optimistic online re-optimization policy

In Reopt, the planner starts with initial policy solution $s^{\star,0}$ of makespan $T^{\star,0} := T(s^{\star,0})$. We use the *-symbol for the policy solution, since *-symbol is reserved for CIOS. Recall that the initial policy solution $s^{\star,0}$ finds an optimal delivery tour under the assumption that all the edges with unknown status are intact. Observe that $T^{\star,0} \leq T^*$. At each occurrence i, when either the truck or the drone discovers a damaged edge in the current truck walk, Reopt immediately updates the current truck-and-drone tour given the current positions of the vehicles; thereby one of the vehicles may still be traversing its edge. Denote the time passed from the beginning of the truck-and-drone tour to the moment of the ith such update as θ_i. We somewhat compromise on notation and denote $s^{\star,i}$ an optimal solution of the following ('remaining') STSP-DI instance: At θ_i, given the current positions of the truck and the drone and assuming all edges with currently unknown status intact, start delivering packages to the remaining delivery addresses while the part of schedule $s^{\star,i}$ traversed up to time θ_i remains fixed. We denote the makespan of this solution as $T^{\star,i} := T(s^{\star,i})$. Obviously, $T^{\star,i+1} \geq T^{\star,i}$.

Theorem 1. *The competitive ratio of the optimistic online re-optimization policy is*

$$\sigma(A^{Reopt}) = \begin{cases} 1 & \textit{if } |V| = 1, \alpha > 1, \\ \geq 2^k & \textit{otherwise.} \end{cases}$$



(a) Policy solution (PS)

figures/reopt_right.pdf

(b) Complete information optimal solution (CIOS)

Figure 6: An unfortunate example for Reopt with $k = 2$ and $|V| = 3$
The truck in PS delivers to address v_2 and v_3 along the lower loop to the right of the depot. It encounters damaged edge (f_1, g_1) at the end of its travel and decides to drive to the depot along the upper loop. But because of another damaged edge (f_2, g_2), it has to travel all the way back.

Proof. For $|V| = 1$ and $\alpha > 1$, the policy will send the drone to deliver, since the drone is α times faster than the truck: If the drone's shortest driving time to the single location to be served is T^*, then the truck would need the larger time $T^* \cdot \alpha$. The competitive ratio is 1 since PS and CIOS coincide.

As for the remaining cases, we start with examining the case of $|V| = 3$ and $\alpha > 0$.

Figure 6a illustrates PS for an unfavorable instance with $V = \{v_1, v_2, v_3\}$ and $k = 2$.

Several loops start and end at the depot v_0. Node v_1 is positioned in the middle of the left loop of length $\alpha \cdot T'$. Nodes v_2 and v_3 are located on the right lower loop from the depot, on opposite sides of the loop's midpoint at a distance of ξ from the midpoint, where $\xi > 0$ is a *very small number*. The length of this loop is T', which equals the truck travel time as the truck speed is normalized to 1.

In total, there are k loops on the right side of the depot, each having a damaged edge $(f_i, g_i), i \in [1, k]$, located close to depot v_0, such that the truck travel time from f_i over g_i to v_0 would be ξ_i in case of no damage. We assume that the distances to the depot decrease in a geometric progression and define $\xi_i := 2^{1-i}\xi$ for $i \geq 1$. The ith loop shares the part from depot v_0 via g_{i-1} to f_{i-1} with loop $i-1$ and leads over f_i and g_i to the depot, for all $i \geq 2$. Further assume that the truck travel time on each loop $i \geq 2$, starting and finishing in the depot if all the edges were intact, is $2^{i-2}T' - \delta_i$ and $\delta_i > 0$ are monotonically increasing. Recall that the truck travel time is T' for the first loop.

If ξ is sufficiently small (see Online Supplement), then in a complete information solution (CIOS), the drone delivers to v_1 and the truck delivers to v_2 and to v_3, taking the lower half-loop there and back (Figure 6b). The makespan of CIOS equals the truck driving time: $T^* = T' + 2\xi$.

Since Reopt treats all the unvisited edges as intact, the truck in PS, which has incomplete information, will decide to take the lower loop on the right side of the depot with a travel time T'. However, it encounters a damaged edge (f_1, g_1) at the end of its travel. Facing two alternatives, to return back taking time $T' - \xi$ or taking the next upper loop (loop $i = 2$) from f_1 via (f_2, g_2) to v_0 of duration $T' - \xi - \delta_2$. The naive truck in PS will choose the latter. Indeed, at the time of this decision, the truck is aware of the damaged edge (f_1, g_1), but does not know that edge (f_2, g_2) is damaged. At each point f_i, $1 \leq i < k$, the truck chooses loop $i + 1 \leq k$ to return to the depot. The travel time from f_i via f_{i+1} to v_0 is $2^{i-1}T' - \delta_{i+1} - \xi_i$, which is the length of loop $i + 1$ minus ξ_i. The travel time of the truck from f_i back to the depot using loops $(i, i-1, \ldots 1)$ is

$$\sum_{j=2}^{i} 2^{j-2}T' + T' - \sum_{j=2}^{i} \delta_j - \left(2\sum_{j=1}^{i-1} \xi_j + \xi_i\right) = 2^{i-1}T' - \sum_{j=2}^{i} \delta_j - \left(2\sum_{j=1}^{i-1} \xi_j + \xi_i\right) \tag{1}$$

If we define $\delta_2 = \xi$ and $\delta_{i+1} = \left(\sum_{j=2}^{i} \delta_j + 2\sum_{j=1}^{i-1} \xi_j\right) + \xi$ for $i \geq 2$, (observe that we added term ξ to avoid ties) then in PS the truck needs more time (by ξ) to drive the whole way back than to take the next upper loop to reach the depot and, thus, prefers the latter.

If we replace the δ_i in the recursive definition of the length of the ith loop, we obtain for $i \geq 2$:

$$2^{i-2}T' - \delta_i = 2^{i-2}T' - \sum_{j=1}^{i-2} 2^{i-j-1}\xi_j - 2^{i-2}\xi \tag{2}$$

Recall that $\xi_i = 2^{1-i}\xi$ and that $\sum_{j=1}^{i-2} 2^{i-j-1}\xi_j$, converges to the infinite sum of a geometric progression. We get $\sum_{j=1}^{i-2} 2^{i-j-1}\xi_j = \sum_{j=1}^{i-2} 2^{i-j-1+1-j}\xi = 2^i\xi\sum_{j=1}^{i-2} 2^{-2j} = 2^i\xi\sum_{j=1}^{i-2}\frac{1}{4^j} \leq 2^i\xi\frac{4}{3} = 2^{i-2}\xi\frac{16}{3}$, which leads to the following lower bound on the length of the ith loop, see (2):

$$2^{i-2}T' - \delta_i \geq 2^{i-2}T' - 2^{i-2}\frac{16}{3}\xi - 2^{i-2}\xi = 2^{i-2}\left(T' - \frac{19}{3}\xi\right) \tag{3}$$

In the worst case, in PS the total truck travel time to reach f_k from depot v_0 taking loops $1, 2, \ldots, k$ as well as the time to travel all the way back to the depot, as stated in (1), adds up to:

$$2 \cdot \left(2^{k-1}T' - \sum_{j=2}^{k}\delta_j - \left(2\sum_{j=1}^{k-1}\xi_j + \xi_k\right)\right) = 2 \cdot \left(2^{k-1}T' - \delta_{k+1} + \xi(1 - 2^{1-k})\right) \geq$$
$$2^k\left(T' - \frac{19}{3}\xi\right), \tag{4}$$

thereby we used the definition of δ_{i+1} and of ξ_i for $i = k$ for the first transformation and the lower bound on the length of the ith loop computed in (3). Recalling $T^* = T' + 2\xi$, observe that this bound approaches 2^kT^* for ξ converging to 0. We conclude that $\sigma(A^{Reopt}) \geq 2^k$, since it is an open question whether Reopt may perform even worse in some other instances. Also observe, that the bound is valid both for the cases of a fast drone with $\alpha \geq 1$ and for the cases of a slow drone with $0 \leq \alpha < 1$.

If $|V| > 3$ (and any $\alpha > 0$), we can construct a similar instance by putting nodes $v_j, j > 3$, in the location of v_2 as an agglomeration.

If $|V| = 2$ and $0 \leq \alpha < 1$, we can straightforwardly adjust the above instance and drop v_1.

In the remaining cases of ($|V| = 2$ and $\alpha \geq 1$) as well as ($|V| = 1$ and $\alpha \leq 1$), we rely on our technical assumption that in case of ties, PS always makes the worst possible decision. To construct an unfortunate instance, we drop v_3 from the example in Figure 6, move v_2 exactly *in the middle* of the right lower loop and set $T' = T^*$. In case of $|V| = 1$ and $\alpha \leq 1$, we also remove node v_1; observe that the truck performs only one delivery in PS. When the truck in PS reaches node v_2, it has a choice between two paths to the depot of the same length $\frac{T'}{2}$, either taking the same way back which is known for sure to be intact, or traveling along the uncertain upper half of the lower right loop. Because of our technical assumption on the ties, the truck decides for the latter. The remaining proof proceeds along the same lines as above. In Appendix A, we establish a lower bound on the competitive ratio that does not rely on the technical assumption about the ties (but for a very special case of $|V| = 1$ and $\alpha = 1$) and show that also this lower bound increases exponentially in k. □

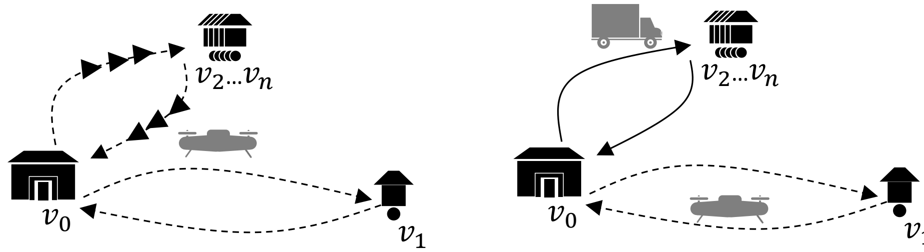
5.2 Analysis of the conservative delivery policy

In CD the drone has to deliver to all addresses in V starting from the depot. We refer to the CD algorithm as A^{CD} and to the competitive ratio of the conservative delivery policy as $\sigma(A^{CD})$.

CD performs badly if there is an agglomeration of many addresses at a long distance from the depot, while there exists a passable path for the truck to these addresses, as we discuss in Lemma 2 and 3.

Lemma 2. $\sigma(A^{CD}) \geq 1 + \frac{|V|-1}{\alpha}$.

Proof. The case of $|V| = 1$ is trivial. If $|V| \geq 2$, consider the example of Figure 7, where v_1 is located such that in CIOS, the drone delivers to v_1 in a return flight from the depot in T^* time, and the truck delivers to an agglomeration of $|V| - 1$ addresses in T^* time (cf. Lemma 1). In PS, the drone performs all $|V|$ deliveries in a return flight from the depot in $T^* + (|V| - 1) \cdot \frac{T^*}{\alpha}$ time, so that the competitive ratio $\sigma(A^{CD})$ cannot be lower than $1 + \frac{|V|-1}{\alpha}$. □



(a) Policy solution (PS) (b) Complete information optimal solution (CIOS)

Figure 7: An unfortunate example for CD with $|V| = n \geq 2$
The drone in PS delivers to each address in V in a separate return flight from the depot. In CIOS, the truck can deliver to the agglomeration of $|V| - 1$ addresses in a single return trip, whereas the drone visits the remaining address v_1.

(a) Policy solution (PS) (b) Complete information optimal solution (CIOS)

Figure 8: An unfortunate example for CD with $|V| = n \geq 1$
The drone in PS delivers to each address in V in a separate return flight from the depot. The truck in CIOS delivers to the agglomeration of $|V|$ addresses in a single return trip.

Lemma 3. $\sigma(A^{CD}) \geq \frac{|V|}{\alpha}$.

Proof. In the example in Figure 8, the truck in CIOS delivers to an agglomeration of $|V|$ addresses in T^* time. In PS, the drone performs all $|V|$ deliveries in a return flight from the depot in $|V| \cdot \frac{T^*}{\alpha}$ time, so that the competitive ratio cannot be lower than $\frac{|V|}{\alpha}$. □

Theorem 2. *The competitive ratio of the conservative delivery policy is* $\sigma(A^{CD}) = \frac{|V|+\max\{\alpha-1,0\}}{\alpha}$.

Proof. Consider two cases: $\alpha \leq 1$ and $\alpha > 1$.

The case of a slow drone with $\alpha \leq 1$ is quite obvious. Since each address is visited by at least one of the vehicles in CIOS, the drone can follow the complete CIOS' tour of this vehicle from the depot to this delivery address and back to the depot. This results in a solution with a makespan of at most $\frac{|V| \cdot T^*}{\alpha}$, since the visit of each delivery address takes at most $\frac{T^*}{\alpha}$ time.

The case of a fast drone with $\alpha > 1$ is more complicated. We will show how to construct a feasible delivery plan s', in which only the drone performs deliveries and the truck never leaves the depot. The makespan of CD obviously cannot be larger than the one of s'. In s' for each $v \in V$, the drone performs one operation that starts and ends at the depot of the following form:

- If v is delivered by the truck in CIOS, then the corresponding delivery tour in s' follows the complete truck walk of the CIOS solution s^* in $T^* - \sum_{o=1}^{n_o} (\tau_o - \tau_o^t)$ time, where $n_o := n_o(s^*)$ is the total number of operations in s^* and the expression on the right-hand side sums up the truck waiting time for the drone in all of these operations. We recall that τ_o is the duration of operation o in s^* given as the maximum of the corresponding truck- and drone time respectively: $\tau_o = \max\{\tau_o^t, \tau_o^d\}$. Since the drone is α times faster than the truck, the time it needs to perform this delivery tour for v in s' is

$$\frac{1}{\alpha}(T^* - \sum_{o=1}^{n_o}(\tau_o - \tau_o^t)). \tag{5}$$

- If for some operation $o_v := o_v(s^*)$, v is delivered by the drone in CIOS and then the delivery tour for v in s' follows the complete truck walk of s^* like above, except for o_v, where it follows the drone walk of the operation instead of the truck walk and still requires at most τ_o time. Thus, the time the drone needs to perform this delivery tour for v in s' is at most

$$\frac{1}{\alpha}(T^* - \sum_{o=1}^{n_o}(\tau_o - \tau_o^t)) + \tau_{o_v} - \frac{\tau_{o_v}^t}{\alpha}. \tag{6}$$

In the example of Figure 3, node v_2 is delivered by the truck in CIOS and the corresponding delivery tour of the drone in s' is $(v_0, f_1, v_2, f_1, v_0)$. The delivery node v_1 is covered by the drone in CIOS, thus, the corresponding drone walk in s' is $(v_0, f_1, v_1, v_2, f_1, v_0)$.

Considering that the number of operations n_o in s^* corresponds to the number of drone delivery nodes in this solution, the makespan of the solution s' sums up to:

$$T(s') \leq |V| \cdot \frac{1}{\alpha}(T^* - \sum_{o=1}^{n_o}(\tau_o - \tau_o^t)) + \sum_{o=1}^{n_o}(\tau_o - \frac{\tau_o^t}{\alpha}) \leq \frac{|V|}{\alpha} \cdot T^* - \sum_{o=1}^{n_o} \frac{(\tau_o - \tau_o^t)}{\alpha} + \sum_{o=1}^{n_o}(\tau_o - \frac{\tau_o^t}{\alpha}) \tag{7}$$

where the last inequality follows from $|V| \geq 1$. Given that the makespan of CIOS is at least the total duration of its operations, i.e., $T^* \geq \sum_{o=1}^{n_o} \tau_o$, that $\alpha > 1$, and that we can eliminate terms $\sum_{o=1}^{n_o} \frac{\tau_o^t}{\alpha}$ in the last two summands, we conclude that

$$T(s') \leq \frac{|V|}{\alpha} \cdot T^* + \frac{\alpha - 1}{\alpha} T^* = \frac{|V| + \alpha - 1}{\alpha} \cdot T^*. \tag{8}$$

Given the makespans of the constructed drone-only delivery plans for both cases of $\alpha \leq 1$ and $\alpha > 1$, as well as given Lemmas 2 and 3, and from the definition of the competitive ratio, we conclude that $\sigma(A^{CD}) = \frac{|V| + \max\{\alpha - 1, 0\}}{\alpha}$. □

5.3 Analysis of the surveillance-first policy

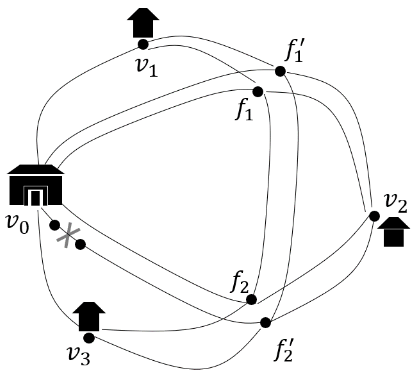
A policy solution (PS) of surveillance first (SF) is divided into a surveillance phase and a delivery phase. Consequently, we will note in the following the route and makespan of these phases with superscripts s and d, respectively. In SF, the planner starts with initial delivery plan $s^{\star,d,0}$ of makespan $T^{\star,d,0} := T(s^{\star,d,0})$. Solution $s^{\star,d,0}$ is an optimal solution of the respective instance if all the edges with unknown status are intact. Observe that $T^{\star,d,0} \leq T^*$. The drone examines all the edges in the truck walk of $s^{\star,d,0}$ and returns to the depot before deliveries can start. In order to examine the truck walk, the drone need not follow it, it just has to visit adjacent nodes of the relevant edges. If the drone returns to the depot after having discovered no damaged edges at time $T^{\star,s,0}$, then the makespan of the policy solution is $T^{\star,s,0} + T^{\star,d,0}$. At each occurrence $i \in \mathbb{N}$, when the drone discovers a damaged edge in the current truck walk, SF updates the current truck-and-drone tour to $s^{\star,d,i}$ and the drone has to inspect the (remaining) edges with unknown status in the truck walk of the new delivery plan $s^{\star,d,i}$ in the surveillance phase. Obviously, $T^{\star,d,i+1} \geq T^{\star,d,i}$. The overall makespan of PS in SF sums up from the surveillance time after k discovered damaged edges, which is $T^{\star,s,k}$, and delivery time $T^{\star,d,k}$.

Since the drone is usually faster than the truck, the main key to the success of SF is to take a short amount of extra time to examine the truck walk with a drone *before* starting the deliveries.

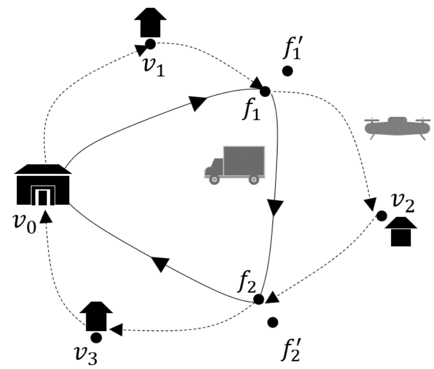
When we think about disaster relief, the decision to pause deliveries until more information on road conditions is collected is always difficult since every minute counts. Thus, the surveillance-first policy seems overly conservative and its benefits are not quite intuitive at first glance. However, in the instance described in Figure 10, the surveillance-first policy performs surprisingly well for a fast enough drone. In particular, we benefit greatly from discarding some impassable paths early on because the detour is quite long. For instance, if the truck notices edge damage at node l_1, it has to travel the whole way back to the depot. Overall, our analysis shows that the surveillance time in SF may be quite small if the visibility of the roads is excellent and the drone can make shortcuts to examine the edges in the planned truck walk.

In Lemma 4, we prove that $\sigma(A^{SF}) \geq \frac{k+1+\alpha}{\alpha}$ for $|V| > 2, \alpha > 1$, which is a quite practical scenario with a fast drone and several delivery addresses. In Appendix B, we show that the same lower bound holds for the remaining cases of $|V| = 2, \alpha > 1$ and $|V| \geq 2, \alpha \leq 1$, if we assume that the policy always takes the worst possible decision in case of ties. Theorem 3 says that the provided lower bound on the competitive ratio is tight.

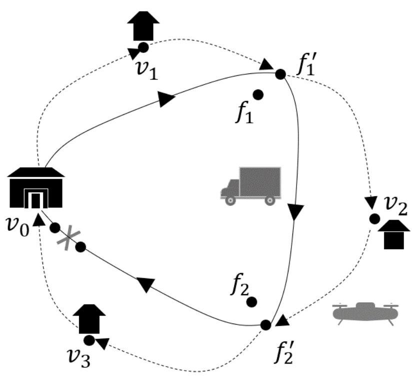
We analyze the importance of drone surveillance in more depth in Theorem 4. According to this theorem, any policy that does not include surveillance detours of the drone has a worse competitive ratio than SF for a range of practice-relevant parameter values, such as $k > 1$ (several damaged edges), $\alpha > \frac{k+1}{k-1}$ (the drone is faster than the truck) and $|V| > (k + 1 + \alpha)$ (many delivery addresses).



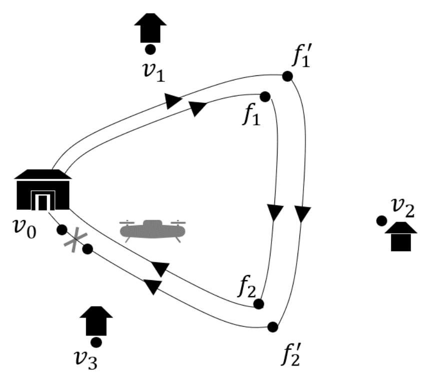
(a) Instance



(b) Complete information optimal solution (CIOS)



(c) Initial delivery plan $s^{*,0}$



(d) Complete surveillance tour in the policy solution

Figure 9: An unfortunate example for SF with $|V| = 3, k = 1$ and $\alpha > 1$
In CIOS, the truck takes the undamaged circular path from the depot and launches the drone from its nodes for each delivery. Since the circular paths are indistinguishable for the online policy, SF proposes at each update of the delivery plan a truck walk through a damaged circular path, until all damages are detected. Since damages are detected only at the end of such path, the drone traverses all circular paths in the surveillance phase of SF, which results in a distance of $(k + 1)$ times the truck walk in CIOS.

Lemma 4. *If $|V| \geq 3$ and $\alpha > 1$, the competitive ratio of SF $\sigma(A^{SF}) \geq \frac{k+1+\alpha}{\alpha}$.*

Proof. Consider instances with $|V| \geq 3$ delivery nodes as illustrated in Figure 9 for $|V| = 3$. There are $k + 1$ *edge- and node-disjoint* cycles, which intersect only at the depot node. On each cycle there are $|V|$ equidistant nodes, namely the depot v_0 and $(|V| - 1)$ Steiner nodes from $D \setminus V$. Between any pair of adjacent nodes of a cycle there is a path (*spike*) connecting these adjacent nodes and leading over exactly one of the nodes from V. We call the structure formed by one cycle together with its spikes a *circular path with spikes (CPS)*. Figure 9b illustrates one such CPS – it is the CPS which the vehicles traverse in CIOS. Observe, the vehicles do not have to visit the $(|V| - 1)$ Steiner nodes of a CPS, but may use them for the drone's launch or landing. In the cycle over the Steiner nodes, starting and ending in the depot of a CPS p, (in Figure 9b these are edges $(v_0, f_1), (f_1, f_2)$ and (f_2, v_0)) the travel distances between consecutive Steiner nodes $c^t(f^p_{i-1}, f^p_i)$ are all the same for all p and all $i \in [1, |V|]$, where the depot $v_0 \in D$ is denoted as f^p_0 and $f^p_{|V|}$, for all p, for convenience. Recall that the edges connecting Steiner nodes and delivery nodes form *spikes*, such that each delivery node $v_i \in V$ is a neighbor of the two respective successive Steiner nodes f^p_{i-1} and f^p_i of each CPS p. The travel distances are $c^t(f^p_{i-1}, v_i) = c^t(v_i, f^p_i) = \frac{\alpha}{2} \cdot c^t(f^p_{i-1}, f^p_i)$. Figure 9 illustrates all $(k + 1)$ CPSs for an instance with $|V| = 3$ and $k = 1$. In k of the CPSs, we have a damaged edge *close* to the depot v_0, which cannot be detected from v_0.

Obviously, no feasible delivery tour can have a makespan of less than $T' = |V| \cdot c^t(f_{i-1}, f_i)$, which is the minimum time to visit all the delivery nodes by the fastest vehicle, the drone. The solution depicted in Figure 9b is CIOS with makespan $T^* = T'$. In this solution, the truck travels the undamaged circular path $\pi^t = (v_0, f_1, f_2, ...f_{|V|-1}, v_0)$ taking time T' and the drone travels along the spikes of this circular path as $\pi^d = (v_0, v_1, f_1, v_2, f_2, ..., v_{|V|}, v_0)$ taking the same time T'. In other words, the drone delivers all the addresses by picking packages from the truck in the Steiner nodes. Note that the vehicles reach the Steiner nodes at the same time and do not wait for each other.

Since SF does not know about road damages, the initially planned truck walk in $s^{*,d,0}$ takes one of the damaged circular paths to launch and retrieve the drone. In the surveillance phase, the drone traverses the complete circular path in $\frac{|V|\cdot c^t(f_{i-1},f_i)}{\alpha} = \frac{T^*}{\alpha}$ time to discover the damage only at the very end of this walk. At each discovery of a damaged edge at the end of a circular path, SF suggests a new delivery route for the truck along yet another damaged path, until all k damaged edges have been discovered. Thus, in the surveillance phase, the drone traverses all $k+1$ circular paths in $T^{*,s,k} = (k+1)\cdot\frac{T^*}{\alpha}$ time. The makespan of PS sums up this surveillance time and the duration $T^{*,d,k} = T^*$ of the final delivery tour. We conclude that $\sigma(A^{SF}) \geq \frac{k+1+\alpha}{\alpha}$. □

Theorem 3. *The competitive ratio of SF is*

$$\sigma(A^{SF}) = \begin{cases} 1 & \text{if } |V| = 1, \alpha > 1 \\ \frac{k+1+\alpha}{\alpha} & \text{otherwise} \end{cases}.$$

Proof. If $|V| = 1$ and $\alpha > 1$, the drone in PS will perform the delivery and $\sigma(A^{SF}) = 1$.

Lemma 4 and Appendix B prove that $\sigma(A^{SF}) \geq \frac{k+1+\alpha}{\alpha}$ if either $|V| > 1$ or $\alpha \leq 1$. It remains to show that $\sigma(A^{SF}) \leq \frac{k+1+\alpha}{\alpha}$.

Observe that the makespan of the delivery schedule in SF cannot exceed the objective value of CIOS: $T^{\star,d,i} \leq T^*$, because SF treats all the edges with not yet known status as intact when planning deliveries. Assume the drone in PS would finish surveillance and return to the depot (if all the remaining edges were intact) at time $T^{\star,s,i-1}$, but it discovers a damaged edge at time $\theta_i \leq T^{\star,s,i-1}$. In SF, the drone selects a shortest flight to investigate the not yet surveyed edges in the truck walk of the updated delivery plan $s^{\star,d,i}$. This cannot take more time than the following feasible walk: Approach the depot by completing the current surveillance flight at time $T^{\star,s,i-1}$ and follow the updated truck walk of $s^{\star,d,i}$ to reach the depot no later than time $T^{\star,s,i-1} + \frac{T^*}{\alpha}$. As the drone starts information collection at time 0, its initially planned duration cannot exceed $T^{\star,s,0} \leq \frac{T^*}{\alpha}$. Moreover, the drone performs at least 1 and at most $k+1$ such surveillance iterations: A damaged edge may be discovered in each of k consecutive surveillance iterations and the drone will make one more iteration to confirm that all the edges in the currently planned truck walk are intact. Consequently, the drone in PS will complete information collection and return to the depot no later than $T^{\star,s,k} \leq (k+1)\frac{T^*}{\alpha}$. Thus, we get an upper bound for $\sigma(A^{SF})$ by summing the received upper bound on the surveillance time $\frac{k+1}{\alpha}\cdot T^*$ and the upper bound on the delivery time T^*, and dividing this sum by T^*. □

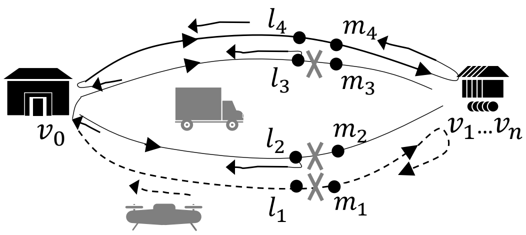
Theorem 4. *For $k > 1$, $\alpha > \frac{k+1}{k-1}$, and $|V| > (k+1+\alpha)$, any truck-and-drone delivery policy in which the drone performs no surveillance at all has a worse competitive ratio than the surveillance-first policy.*

Remarks: We define the phrase "the drone performs no surveillance at all" as follows: The drone performs deliveries from the truck to some addresses $v \in V$ by moving along the shortest path to the respective address v and returning along the shortest path to the truck. In other words, the drone may only collect information on the damaged edges as a 'by-product', i.e, if an adjacent node of such an edge lies on the shortest delivery path of the drone anyway. We have already proved that the competitive ratio of SF is $1 + \frac{k+1}{\alpha}$ if either $|V| > 1$ or $\alpha \leq 1$. Thus, we only need to find an instance I with a larger ratio $\frac{A(I)}{OPT(I^*)}$ for any policy (algorithm) A without drone surveillance.

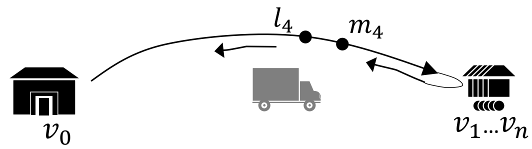
Proof. For any $k > 1$, $\alpha > \frac{k+1}{k-1}$, and $|V| > (k+1+\alpha)$, we can construct an example similar to that in Figure 10 with lookout nodes $l_j \in L \setminus D$ and $m_j \in L \setminus D$, and edges $(v_0, l_j), (l_j, m_j), (m_j, v_i)$ for $j \in [1, k+1], v_i \in V$. All delivery nodes belong to the same agglomeration. Some edge $(l_{k'}, m_{k'}), k' > 1$, is intact and $(k-1)$ remaining edges $(l_j, m_j), k > 1, k \neq k'$, as well as edge (l_1, m_1) are damaged, but this information is initially unknown to the decision maker.

Paths $(v_0, l_j, m_j, v_i),\ j > 1, v_i \in V$, have equal lengths of $\frac{T'}{2}$. whereas the length of paths (v_0, l_1, m_1, v_i) is $\frac{T'}{2} - \epsilon$. For this purpose, we set the first edge $c^t(v_0, l_1)$ to be infinitesimally smaller by $\epsilon > 0$ than $c^t(v_0, l_j)$ for $j > 1$, whose edge lengths are all equal. As for the remaining edges, we keep the lengths of (m_j, v_i) equal and the lengths of (l_j, m_j) equal for any $j \in [1, k+1], v_i \in V$.

The makespan of CIOS is $T^* = T'$, provided ϵ is sufficiently small to make the drone-only deliveries unattractive, e.g., $\epsilon < T'\frac{k+1}{2(k+1+\alpha)}$. Indeed, in a feasible solution, the truck delivers to all addresses $v_i \in V$, using the undamaged path $(v_0, l_{k'}, m_{k'}, v_1)$ from the depot there and back within the time of $T^* = T'$. Any feasible solution, in which the truck



(a) A possible policy solution (PS) without surveillance



(b) A possible complete information optimal solution (CIOS)

Figure 10: An example of the benefits of drone surveillance with $k = 3$
It is important to discard some impassable paths early on due to the lengthy detours they would otherwise cause for the truck.

visits one or several nodes $v_i \in V$, has the makespan of at least T'. In the remaining solutions, the drone delivers to all $v_i \in V$ starting from v_0, because no launch is possible in lookout nodes l_j and m_j. The makespan of such drone-only deliveries equals $|V| \cdot \frac{T'-2\epsilon}{\alpha} > (k+1+\alpha) \cdot \frac{T'-2\epsilon}{\alpha} > (k+1+\alpha) \cdot \frac{T'}{\alpha} - T' \cdot \frac{k+1}{\alpha} = T'$; here we used relations $|V| > (k+1+\alpha)$ (from the definition of the theorem) and $\epsilon < T' \frac{k+1}{2(k+1+\alpha)}$.

In PS, the drone may start or end its delivery operations only in nodes v_0 and $v_i \in V$, because l_j's and m_j's are lookout nodes ($l_j, m_j \in L \setminus D, \forall j \in [1, k+1]$). If the drone launches in v_0 to deliver to some node $v_i \in V$, then it always takes the shortest path (v_0, l_1, m_1, v_i). The same is true (in the reverse direction), if the drone returns to the depot v_0 after delivering to v_i . The case is obvious, if the drone delivers to v_i using some node $v_{i'} \in V$, as a launch or return node. To sum up, the drone will not collect information on the damaged edges $(l_j, m_j), j > 1$, in policies without drone surveillance.

First, we consider delivery policies where the truck performs at least one delivery. Since paths (v_0, l_j, m_j, v_i) are identical for $j > 1, v_i \in V$, no delivery policy can distinguish between them. Therefore, for any such delivery policy, we can construct an instance with such damaged edges that the truck has to perform $(k-1)$ detours before it can deliver packages using the intact path $(v_0, l_{k'}, m_{k'}, \ldots), k' \in [2, k+1]$. We let the length of each detour (v_0, l_j, v_0) be δT^* for some $0 < \delta < 1$ and $j > 1$. Then, the truck may require at least $(\delta(k-1)+1)T^*$ units of time to deliver its package(s). By shifting edges (l_j, m_j) to the right so that $1 > \delta > \frac{k+1}{(k-1)\alpha}$, we obtain a worse ratio than we do for the surveillance-first policy.

Now, consider delivery policies without surveillance, in which the truck does not perform any deliveries and does not visit any $v_i \in V$. As reasoned above, in this case the drone can only launch from node v_0 for its delivery operations. Since $|V|$ is sufficiently large, i.e., $|V| > (k+1+\alpha)$, the resulting ratio for these drone delivery policies is worse than for the surveillance-first policy, see Theorem 3. □

6 Discussion and Outlook

In this paper, we investigate the Steiner traveling salesman problem with a truck and a drone with incomplete information (STSP-DI) motivated by deliveries of emergency supplies using a truck and a drone under unknown road conditions, which are often faced by disaster relief teams. The paper computes competitive ratios for several alternative delivery policies for the STSP-DI, including the widespread online re-optimization policy (Reopt). Competitive ratios reveal the worst-case performance of these policies compared to the baseline of an optimal policy for the case of complete information about the road conditions. Competitive analysis is especially relevant in the context of disaster relief, where the worst outcomes have to be avoided. We conduct parametric analysis and differentiate problem classes according to the number of damaged edges $k \in \mathbb{N} \cup \{0\}$, the factor by which the drone is faster than the truck $\alpha \in \mathbb{R}^+$ as well as the number of required deliveries $|V| \in \mathbb{N}$.

Our analysis reveals that Reopt performs poorly in realistic worst-case instances if several damaged edges are present, moreover, its competitive ratio increases exponentially with k. Reopt fails if several alternative routes to a required destination exist and all but few routes are impassable, so that the truck gets easily trapped in a damaged region. This situation is typical in inundated urban areas, for instance. Our analysis underscores the importance of a rather counter-intuitive decision – to delay the start of emergent deliveries, so that the drone can examine the most relevant road segments for the truck tour. A straightforward implementation of such a policy, surveillance first (SF), has a much better competitive ratio than Reopt if the road network is severely damaged, i.e., if k is large enough. Although the question about a policy with the best possible competitive ratio remains open, we narrow its properties analytically and prove that such policy should necessarily include detours for the purposes of surveillance. More specifically, no policy that does not consider surveillance detours can be better than SF in terms of the competitive ratio if the drone is sufficiently fast ($k > 1$ and $\alpha > \frac{k+1}{k-1}$) and the number of the required deliveries is sufficiently high ($|V| > (k + \alpha + 1)$). We also quantify the competitive ratio for the conservative delivery policy, in which the drone performs all the deliveries in return flights from the depot.

A number of questions remain for future research. First of all, both the Reopt and the SF rely on re-optimization, in which an NP-hard truck-and-drone routing problem, which is the full-information counterpart of the STSP-DI, is solved repeatedly. Therefore, can we find fast heuristics suitable for the online (adjustable) planning scenario? Secondly, future projects may elaborate on a policy with the best competitive ratio. Thirdly, future research should investigate extensions and variations of the STSP-DI, including the influence of vehicle fleet characteristics (e.g., the cases of several drones and/or several vehicles), of the energy capacity limits of the drone's battery, of information transmission mechanisms among the vehicles as well as of deadlines/releases and priorities of the deliveries.

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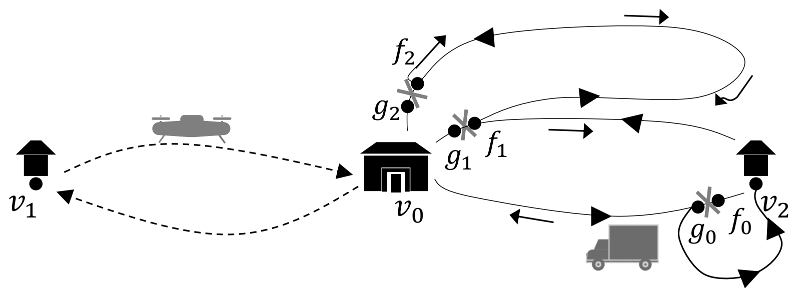
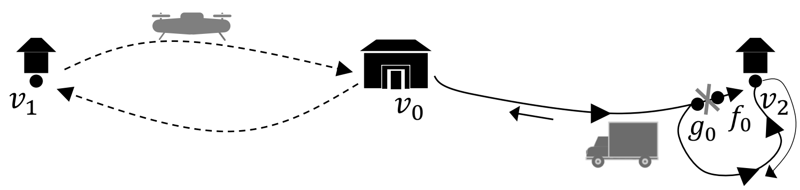
Appendix A: Further unfortunate examples for Reopt

If either ($|V| = 2$ and $\alpha \geq 1$) or ($|V| = 1$ and $\alpha \leq 1$), the proof of $\sigma(A^{Reopt}) \geq 2^k$ in Theorem 1 relies on our technical assumption that if the truck in the PS faces alternative paths of the same length, it always chooses the worst possible path. In particular, it prefers a not yet completely investigated path to the path of the same length known to be intact. This assumption may appear counterintuitive. Therefore, we design further unfavorable examples for Reopt in this appendix, which do not rely on the technical assumption above and they also result in a lower bound on the competitive ratio that is exponential in k. The example in Lemma 5 covers the case of ($|V| = 2$ and $\alpha \geq 1$) and the one in Lemma 6 covers the case of ($|V| = 1$ and $\alpha < 1$), respectively.

Lemma 5. *For any $|V| = 2$ and $\alpha \geq 1$, the competitive ratio of the optimistic online re-optimization policy is $\sigma(A^{Reopt}) \geq 2^{k-1}$.*



(a) Policy solution (PS)



(b) Complete information optimal solution (CIOS)

Figure 11: An unfortunate example for Reopt with $k = 3$ and $|V| = 2$
The truck in PS plans to deliver to address v_2 along the lower loop via g_0 and f_0 to the right of the depot. It encounters damaged edge (f_0, g_0) close to v_2 and takes a detour. At v_2, the truck decides to return to the depot along the yet unexplored upper part of the right lower loop of length $\frac{T'}{2}$, since it would take $\frac{T'}{2} + \xi, \xi > 0, \xi \to 0$, to return the same way back to the depot. At the end of its travel, the truck encounters damaged edge (g_1, f_1) and decides to drive to the depot along the upper loop. But because of another damaged edge (f_2, g_2), it has to travel all the way back.

Proof. We *relax* the assumption that PS always chooses the worst possible path in case of a tie.

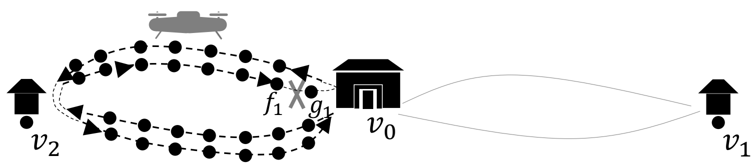
The case of $k = 1$ is trivial. In the following, we will discuss two cases: ($k \geq 2$, $\alpha \neq 1$) and ($k \geq 2$, $\alpha = 1$).

For $k \geq 2$ and $\alpha \neq 1$, consider the instance in Figure 11. Nodes v_1 and v_2 are located at the mid-points of two loops: the left loop (of length $\alpha T'$) and the lower right loop (of length T'), respectively. The distances are selected such that the truck and the drone would complete their trips at the same time through the left and lower right loop, respectively, if all edges were intact (cf. Lemma 1). There is a damaged edge (g_0, f_0) in the vicinity of v_2 and there is an additional intact direct edge between g_0 and v_2, which allows for a detour. Let the truck travel time between depot v_0 via g_0 over the detour to v_2 be $\frac{T'}{2} + \xi$, where $\xi > 0$ is very small.

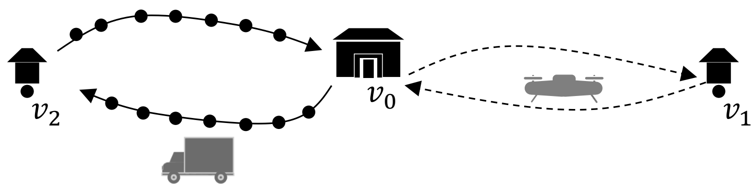
The remaining edges in the instance are constructed as explained in Theorem 1 with the difference that there are just $(k-1)$ loops on the right side of the depot, including the lower right-hand loop. Similar to Theorem 1, each such loop has a damaged edge $(f_i, g_i), i \in \{1, \ldots, k-1\}$, located close to depot v_0, such that the truck travel time from f_i over g_i to v_0 equals ξ. Also here, the ith loop shares the path from depot v_0 via g_{i-1} to f_{i-1} with loop $(i-1)$ and is connected via the edges (f_i, g_i) and (g_i, v_0) to the depot, for all $i \geq 2$.

Obviously, the makespan of CIOS T^* is bounded by the following interval: $T' \leq T^* \leq T' + 2\xi$. Observe that $T' \leq T^*$, since the makespan of CIOS cannot be smaller than the time it takes the fastest vehicle to deliver to v_1 and v_2 and return to v_0 assuming all the edges are intact (cf. Lemma 1). Moreover, in a feasible solution with makespan $T' + 2\xi$, the drone delivers to v_1 and, in parallel, the truck delivers to v_2 taking the lower loop via the direct detour-edge (g_0, v_2) there and back.

In PS, the truck tries to deliver to v_2 traveling the walk $(v_0, g_0, f_0, v_2, f_1, g_1, v_0)$, since it does not know that specific edges are damaged. Because of the damage at (g_0, f_0), the truck takes the detour via the direct edge (g_0, v_2), provided ξ is sufficiently small ($\xi \to 0$). At v_2, it has a choice to travel all the way (v_2, g_0, v_0) back or to proceed along



(a) Surveillance in the policy solution (PS)



(b) Delivery in a policy solution (PS) and complete information optimal solution (CIOS)

Figure 12: An unfortunate example for SF with $k = 1$ and $|V| = 2$
In the initial delivery plan $s^{\star,d,0}$ of PS, the truck travels along the inner loop to the left of the depot. So the drone in PS has to examine the edges of this inner loop, but discovers a damaged edge in the end of this initial surveillance sortie. As a result, the drone in PS has to proceed with surveying the complete replanned truck walk along the outer loop to the left of the depot. Afterward, the truck and the drone can deliver packages. The final delivery plan in PS coincides with CIOS.

(v_2, f_1, g_1, v_0). The latter path takes $\frac{T'}{2}$ and is shorter than the former, which has the duration of $(\frac{T'}{2} + \xi)$. The proof proceeds along the same lines as in Theorem 1.

For $k \geq 2$ and $\alpha = 1$, we have to modify the example. Indeed, otherwise, the Reopt-planner would rather send the truck to the left and the drone to the right, since we already see from the depot that the left loop is intact, but we have uncertainty whether all the edges are intact for the right-hand lower loop, and both loops are of the same length. To prevent this, we must introduce two additional lookout nodes in the left loop such that both loops are indistinguishable for the ex-ante planner. □

Lemma 6. *For any $|V| = 1$ and $\alpha < 1$, the competitive ratio of the optimistic online re-optimization policy is $\sigma(A^{Reopt}) \geq 2^{k-1}$.*

Proof. If $|V| = 1$ and $\alpha < 1$, we simply remove node v_1 in the example in Figure 11. PS remains the same as described in Lemma 5, since initially the truck, as the faster vehicle, will try to deliver to v_2 driving along $(v_0, g_0, f_0, v_2, f_1, g_1, v_0)$.

In CIOS, the makespan remains bounded by the interval $T' \leq T^* \leq T' + 2\xi$. In a feasible solution, the truck delivers to v_2 taking the lower loop via the direct detour-edge (g_0, v_2) there and back, hence, $T^* \leq T' + 2\xi$. Further, the makespan of CIOS cannot be smaller than the time it takes the fastest vehicle to deliver to v_2 in a return trip from the depot v_0 assuming all the edges intact, hence, $T^* \geq \min\{T', \frac{T'}{\alpha}\}\} = T'$. □

Appendix B: Further unfortunate examples for SF

In Lemma 4, we provided a lower bound for the competitive ratio of the surveillance-first policy (SF) in the most relevant case, where the drone is faster than the truck and deliveries have to be made to more than two addresses. In the following, we will provide additional worst case instances for SF in the general case of $|V| > 1$ or $\alpha \leq 1$, which rely on the assumption that the policy always chooses the worst possible path, when it faces alternative paths of the same length.

Lemma 7. *If $|V| > 1$ or $\alpha \leq 1$, the competitive ratio of SF $\sigma(A^{SF}) \geq \frac{k+1+\alpha}{\alpha}$.*

Proof. If $|V| > 1$, there are instances, such as the one presented in Figure 12, where the truck in CIOS will perform at least one delivery notwithstanding the value of parameter α (cf. Lemma 1).

We consider the example in Figure 12 with $k = 1$ and $|V| = 2$ constructed as described in Lemma 1, in which the drone in CIOS needs T^* units of time to deliver to address v_1, and the truck needs T^* units to deliver to address v_2. Furthermore, there are four paths of equal lengths and very low visibility connecting v_0 and v_2. This is achieved by placing many lookout nodes along the paths. Thus, the drone in PS, basically, has to traverse the whole path to check whether it is passable. Since the four shortest paths between v_0 and v_2 have equal lengths, a possible delivery route of the truck in PS will initially go through the paths in the inner loop. Assume there is a damaged edge (f_1, g_1) at the very end of the drone's surveillance flight through the inner loop, such that the drone reaches f_1 at time θ_1 close to $\frac{T^*}{\alpha}$. The truck in the re-planned delivery trip travels along the outer loop. The total surveillance time, including the time to examine edges in the updated truck walk and the delivery time after the surveillance is completed, converges to $\frac{2T^*}{\alpha} + T^*$. Therefore, $\sigma(A^{SF}) \geq \frac{2+\alpha}{\alpha}$ for $k = 1$.

We can construct examples for $k \geq 2$ in a similar manner using $(k + 1)$ low-visibility loops of the same length that start and end at v_0 and where v_2 is in the middle of these loops.

For $|V| = n > 2$, we place all delivery addresses $v \in \{v_2, ..., v_n\}$ at the location of node v_2 as an agglomeration. If $|V| = 1$ and $\alpha \leq 1$, we construct an example where node v_1 is dropped in Figure 12. In case of equal speeds of the truck and the drone ($\alpha = 1$), we use our technical assumption that a possible SF solution will require the truck to perform the only delivery. □

Online Supplement: The makespan of CIOS for the unfortunate example for Reopt in Theorem 1

Let's recall the description of the unfavorable instance with $V = \{v_1, v_2, v_3\}$ and $k \in \mathbb{N} \cup \{0\}$ from Theorem 1, which is illustrated schematically in Figure 6. Nodes v_2 and v_3 are located on the lower right-hand loop from the depot of length T'. They are located on opposite sides at a distance of ξ from the middle of the loop, where ξ is a very small number. Node v_1 is positioned in the center of the left-hand loop. This loop has the length of $\alpha \cdot T'$. There are, in total, k loops to the right side of the depot, each having a damaged edge $(f_i, g_i), i \in [1, k]$, located close to depot v_0, such that the truck travel time from f_i over g_i to v_0 equals ξ. The ith loop shares the path from depot v_0 via g_{i-1} to f_{i-1} with loop $i-1$ and connects through the edges (f_i, g_i) and (g_i, v_0) to the depot, for all $i \geq 2$. Further, assume that the truck travel time on each loop $i \geq 2$, starting and finishing at the depot if all the edges were intact, is $2^{i-2}T' - \delta_i$ and $\delta_i > 0$ are monotonically increasing. Recall that the truck travel time is T' for the first loop.

Lemma 8. *Let an unfavorable instance with $V = \{v_1, v_2, v_3\}$ and $k \in \mathbb{N} \cup \{0\}$ be given as presented in Theorem 1 . Let edges (f_i, g_i) and edges (v_0, g_i) have lengths of $\frac{\xi}{2}$ for $i \in [1, k]$. If $\xi < \min\{\frac{\alpha T'}{4}, \frac{T'}{4\alpha+5}\}$, then the makespan of CIOS is $T' + 2\xi$.*

Proof. Let's prove that the makespan of CIOS equals $T^* = T' + 2\xi$. In one such possible solution, the drone delivers to v_1 and the truck delivers to v_2 and to v_3 taking the lower half-loop there and back (see Figure 6a).

Let's observe that the drone can launch from the truck at nodes v_0, v_1, v_2, v_3, g_i, or f_i, $i \in [1, k]$.

We will prove by enumerating possible feasible solutions and showing that their makespan cannot be lower than $(T' + 2\xi)$. First, observe that any feasible solution, in which the truck delivers to (or simply visits) node v_3 or node f_i cannot have a lower objective value than $T' + 2\xi$. Similarly, no solution, in which the truck delivers to (or simply visits) *both* nodes v_1 and v_2, can have a better makespan either. Indeed, the truck needs at least as much time as it takes to traverse the minimum duration tour from the depot over nodes v_1 and v_2 back to the depot, which equals $\alpha T' + T' - 2\xi > T' + 2\xi$ because $\xi < \frac{\alpha T'}{4}$.

In the remaining cases, the drone necessarily delivers to v_3, the truck does not visit nodes v_3 and $f_i, i \in [1, k]$, and the truck visits at most one node from v_1 and v_2. Let's enumerate the following possibly overlapping cases:

(i) First, consider complete information solutions $s \in G_1$, in which the truck does not visit v_1. Then, the drone has to deliver to v_1 in an operation that has its start and end at nodes that are distinct from v_1. The makespan of the solutions in G_1 cannot be less than the following computation:

- the drone delivers to v_1 in a return flight from v_0 first, which takes time T' (observe that v_0 is the closest node in which the drone can meet the truck to take another package, moreover, any shortest path of the drone from v_1 to $v \in L \setminus \{v_1\}$ contains v_0),
- then the drone travels there and back to v_3 with the *speed of the fastest vehicle*, which cannot take more time than $\min\{T' - 2\xi, \frac{T'-2\xi}{\alpha}\}$.

We obtain that:

$$T(s \in G_1) \geq T' + \min\{T' - 2\xi, \frac{T' - 2\xi}{\alpha}\} > T' + 2\xi. \tag{9}$$

In the last transformation, we used the relation $\xi < \frac{T'}{4\alpha+5}$, which implies that $T' - 2\xi > \xi(4\alpha + 5) - 2\xi$, and the nonnegativity of α and ξ.

(ii) Let the truck visit v_1. If the truck visits v_1, then it cannot visit neither v_2 or v_3. In this case, the makespan is the maximum of $\alpha T'$ (the time that the truck needs to visit v_1 in a return trip from v_0) and $\frac{T'-3\xi}{\alpha} + \frac{T'-2\xi}{\alpha}$

(the time, the drone needs to deliver to v_3 and v_2 from the closest possible launch-return points, which are g_1 and v_0, respectively).

Consider two cases:

- If $\alpha \geq \left(1 + \frac{2}{3}\right)$ and using the fact that $\xi < \frac{T'}{(4\alpha+5)} < \frac{T'}{3}$, then the makespan of the resulting solution cannot be smaller than
$$\max\{\alpha T', \frac{T'-3\xi}{\alpha} + \frac{T'-2\xi}{\alpha}\} \geq \alpha T' \geq T' + \frac{2T'}{3} > T' + 2\xi. \tag{10}$$
- If $\alpha < \left(1 + \frac{2}{3}\right)$ and using the fact that $\xi < \frac{T'}{(4\alpha+5)} < \frac{T'(2-\alpha)}{4\alpha+5}$, then the makespan of the resulting solution cannot be smaller than
$$\begin{aligned}&\max\{\alpha T', \frac{T'-3\xi}{\alpha} + \frac{T'-2\xi}{\alpha}\} \geq \frac{T'-3\xi}{\alpha} + \frac{T'-2\xi}{\alpha} = \\ &T' + \frac{T'(2-\alpha)}{\alpha} - \frac{5\xi}{\alpha} > T' + \frac{(4\alpha+5)\xi}{\alpha} - \frac{5\xi}{\alpha} > T' + 2\xi.\end{aligned} \tag{11}$$

□