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Large sample properties of an IV estimator based on the Ahn and Schmidt moment conditions

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Abstract. We propose an instrumental variables (IV) estimator based on nonlinear (in parameters) moment conditions for estimating linear dynamic panel data models and derive the large sample properties of the estimator. We assume that the only explanatory variable in the model is one lag of the dependent variable and consider the setting where the absolute value of the true lag parameter is smaller or equal to one, the cross section dimension is large, and the time series dimension is either fixed or large. Estimation of the lag parameter involves solving a quadratic equation and we find that the lag parameter is point identified in the unit root case; otherwise, two distinct roots (solutions) result. We propose a selection rule that identifies the consistent root asymptotically in the latter case and derive the asymptotic distribution of the estimator for the unit root case and for the case when the absolute value of the lag parameter is smaller than one.

Keywords. Panel data, linear dynamic model, quadratic moment conditions, instrumental variables, large sample properties.

JEL codes. C23, C26.

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1 Introduction

We propose an IV estimator based on the nonlinear (in parameters) Ahn and Schmidt (1995) moment conditions for the estimation of linear dynamic panel data models and derive the large sample properties of the estimator. We consider a model with first order dynamics and no further explanatory variables besides the lagged dependent variable; we focus on the setting where the absolute value of the lag parameter is smaller or equal to one and where the cross section dimension is large and the time series dimension is either fixed or large. We show that the assumption that there is no serial correlation in the idiosyncratic remainder components – from which the nonlinear moment conditions arise – turns out to have more identifying power than what Anderson and Hsiao (1981) or even Arellano and Bond (1991) estimators suggest. In particular, persistent autoregressive parameters can be identified even for the unit root case. We confront the possibility of multiple roots, which affects the asymptotic analysis. The derivations in this paper show that fixed-T and large-T results are very different (especially for the unit root case) and that inferences obtained from the asymptotic approximations tend to depend on higher-order moments of the idiosyncratic remainder component. Our results indicate the regions of the parameter space and the settings of the data generating process (DGP) for which the nonlinear moment conditions are particularly useful.

The popularity of GMM estimators for estimating linear dynamic panel data models has increased given the availability of software[4] allowing for the use of a set of default linear moment conditions such as those proposed by Holtz-Eakin, Newey, and Rosen (1988) and Arellano and Bover (1995). Ahn and Schmidt (1995) propose the nonlinear moment conditions that also arise from standard assumptions, while Ahn and Schmidt (1997) derive an asymptotically equally efficient linearized GMM estimator. Using nonlinear moment conditions in estimation is not very popular in practice because a closed form solution does not exist and nonlinear optimization techniques are required. Unfortunately, the finite sample performance of the more popular linear GMM estimators may be poor for certain DGPs (for an extensive Monte Carlo study see Kiviet, Pleus, and Poldermans, 2017). As indicated by Monte Carlo evidence of Bun and Sarafidis (2015), however, finite sample performance may improve by quite a large margin, when the nonlinear moment conditions are taken into account.

Bun and Kleibergen (2014) and Gorgens, Han, and Xue (2016) highlight the potential for using

[4]See, e.g., the programs `xtabond2` (Roodman, 2009a) and `xtdpdgmm` (Kripfganz, 2018) for `Stata` and the packages `plm` (Croissant and Millo, 2008), `panelvar` (Sigmund and Ferstl, 2019), and `pdynmc` (Fritsch, Pua, and Schnurbus, 2019) for `R`.

nonlinear moment conditions to deal with identification failures in linear dynamic panel data models that arise from relying on the usual sets of linear moment conditions only. More specifically, Bun and Kleibergen (2014) modify the nonlinear moment conditions to deal with the case where the true value of the lag parameter is unity. They consider the worst case DGPs. Gorgens, Han, and Xue (2016) focus on characterizing the conditions for GMM identification and establish that the nonlinear moment conditions can provide full or partial identification of the lag parameter even when linear moment conditions fail to do so.

Alvarez and Arellano (2003) show that the GMM estimator for linear dynamic panel data models is consistent but has a limiting distribution with a nonzero center when both $n, T \to \infty$ and T/n tends to a nonzero positive constant. The reason for the nonzero center can be traced to instrument proliferation since the moment conditions proposed by Holtz-Eakin, Newey, and Rosen (1988) are of order T^2. Recently, Hsiao and Zhang (2015) show that the original Anderson-Hsiao (1981; 1982) estimator does not have this nonzero center in its limiting distribution. It is of interest to determine the benefits of adding the instruments implied by the Ahn and Schmidt (1995) nonlinear moment conditions.

The paper is structured as follows: Section 2.1 introduces the model structure and the underlying assumptions. Section 2.2 establishes consistency for the unit root case, discusses identification of the consistent root when the absolute value of the lag parameter is smaller than one, and discusses an asymptotically consistent selection rule. Section 2.3 derives the asymptotic distribution for both cases and Section 3 concludes.

2 Main results

2.1 Modeling framework

Consider the linear panel data model with an error term $u_{i,t}$ that exhibits first order dynamics:

$$y_{i,t} = \alpha_i + u_{i,t}, \quad u_{i,t} = \rho u_{i,t-1} + \varepsilon_{i,t}, \quad \rho \in (-1, 1] \,.$$

The equation gives rise to the following more familiar linear dynamic panel data model:

$$y_{i,t} = (1-\rho)\,\alpha_i + \rho y_{i,t-1} + \varepsilon_{it}.$$

The preceding formulation allows the limit theory to be continuous at $\rho = 1$. We make the following assumptions:

A1 The $u_{i,-1}$ represent initial conditions that are independently drawn from some distribution F, where for all i, $\mathrm{E}\left(u_{i,-1}\right)=0$, and $\mathrm{E}\left(u_{i,-1}^{2}\right)<\infty$.

A2 $\varepsilon_{i,t}$ is independent of α_i and $u_{i,-1}$, $\varepsilon_{i,t}\sim$ i.i.d. $\left(0,\sigma_{\varepsilon}^{2}\right)$, with $\mathrm{E}\left(\varepsilon_{i,t}^{3}\right)=\mathrm{E}\left(\varepsilon_{i,1}^{3}\right)<\infty$, and $\mathrm{E}\left(\varepsilon_{i,t}^{4}\right)=\mathrm{E}\left(\varepsilon_{i,1}^{4}\right)<\infty$ for all $i=1,\ldots n$ and $t=0,\ldots,T$.

A3 The α_i are unobservables that are independently drawn from some distribution G for all i, and $\mathrm{E}\left(\alpha_i^2\right)<\infty$.

A4 For asymptotic considerations assume that $n\to\infty$ first, then $T\to\infty$.

Assumptions A1 to A3 effectively confine the analysis to a simple setting of cross sectional independence. Although this may not be a realistic setting in practice, inclusion of time dummies and interactive effects will complicate the analysis given Assumption A4. Some heterogeneity across i or t may be possible but we need Lindeberg-Feller type conditions and some restrictions on the rates of growth of heterogeneity. It is typically easier to derive sequential limits. We have yet to write down the theory under joint limits or at least use the main result in Phillips and Moon (1999) to convert sequential limiting distributions into joint limiting distributions.

We will extensively use second-moment information for the limit theory. According to Han and Phillips (2010), the following holds for the second moments of the dependent variable:

$$\mathrm{E}\left(y_{i,t}y_{i,s}\right)=\mathrm{E}\left(\alpha_i^2\right)+\frac{\sigma_{\varepsilon}^{2}\rho^{|t-s|}}{1-\rho^2},\ |\rho|<1,$$
$$\mathrm{E}\left(y_{i,t}y_{i,s}\right)=\mathrm{E}\left(\alpha_i^2\right)+\mathrm{E}\left(u_{i,-1}^{2}\right)+\sigma_{\varepsilon}^{2}\left(s\wedge t+1\right),\ \rho=1.$$

Consider now the model in first differences, so as to eliminate the fixed effects:

$$\Delta y_{i,t}=\rho\Delta y_{i,t-1}+\Delta\varepsilon_{i,t}.$$

A possible estimator for ρ could be based on the following nonlinear moment condition that arises from the lack of serial correlation in $\varepsilon_{i,t}$ and the lack of correlation between $\varepsilon_{i,t}$ and α_i (see Assumption A2):

$$\mathrm{E}\left[\left(y_{i,T}-\rho y_{i,T-1}\right)\left(\Delta y_{i,t-1}-\rho\Delta y_{i,t-2}\right)\right]=0,\quad t=3,\ldots,T. \tag{1}$$

Instead of using the GMM framework, we resort to using all the nT observations to approximate Equation (1). The usual formulation combines either of the preceding nonlinear moment conditions with other linear moment conditions, like those of Holtz-Eakin, Newey, and Rosen (1988), to form a GMM estimator. In the case we consider here, we have a simple IV estimator where the instruments are of the form $y_{i,t} - \rho y_{i,t-1}$. One thing to note about these instruments is that they are unobservable. As a result, we solve the following equation for ρ to obtain parameter estimates:

$$\frac{1}{n}\sum_{i=1}^{n}\sum_{t=3}^{T}\left(y_{i,T} - \rho y_{i,T-1}\right)\left(\Delta y_{i,t-1} - \rho\Delta y_{i,t-2}\right) = 0. \tag{2}$$

Observe that Equation (2) is quadratic in ρ, as opposed to the usual linear moment conditions which are linear in ρ. To facilitate the asymptotic analysis, we rewrite Equation (2) as:

$$\begin{aligned}
&\frac{1}{n}\sum_{i=1}^{n}\left(y_{i,T} - \rho y_{i,T-1}\right)\sum_{t=3}^{T}\left(\Delta y_{i,t-1} - \rho\Delta y_{i,t-2}\right) = 0 \\
&\frac{1}{n}\sum_{i=1}^{n}\left(y_{i,T} - \rho y_{i,T-1}\right)\left[\sum_{t=3}^{T}\Delta y_{i,t-1} - \rho\sum_{t=3}^{T}\Delta y_{i,t-2}\right] = 0 \\
&\frac{1}{n}\sum_{i=1}^{n}\left(y_{i,T} - \rho y_{i,T-1}\right)\left[\left(y_{i,T-1} - y_{i,1}\right) - \rho\left(y_{i,T-2} - y_{i,0}\right)\right] = 0.
\end{aligned}$$

As a result, Equation (2) can be expressed in the form

$$A_{n,T}\rho^2 + B_{n,T}\rho + C_{n,T} = 0, \tag{3}$$

where

$$\begin{aligned}
A_{n,T} &= \frac{1}{n}\sum_{i=1}^{n} y_{i,T-1}\left(y_{i,T-2} - y_{i,0}\right), \\
B_{n,T} &= -\frac{1}{n}\sum_{i=1}^{n}\left[y_{i,T-1}\left(y_{i,T-1} - y_{i,1}\right) + y_{i,T}\left(y_{i,T-2} - y_{i,0}\right)\right], \\
C_{n,T} &= \frac{1}{n}\sum_{i=1}^{n} y_{i,T}\left(y_{i,T-1} - y_{i,1}\right).
\end{aligned}$$

2.2 Consistency

2.2.1 The case when $|\rho| < 1$

First, we study the large-n limits when $|\rho| < 1$ but $\rho \neq 0$. Since we impose Assumptions A1 to A3, we may apply a standard law of large numbers and conclude that

$$\begin{aligned}
A_{n,T} &\xrightarrow{p} \mathrm{E}\left[y_{i,T-1}\left(y_{i,T-2} - y_{i,0}\right)\right] = \frac{\sigma_\varepsilon^2}{1-\rho^2}\left(\rho - \rho^{T-1}\right) = \frac{\rho\sigma_\varepsilon^2}{1-\rho^2}\left(1 - \rho^{T-2}\right), \\
-B_{n,T} &\xrightarrow{p} \mathrm{E}\left[y_{i,T-1}\left(y_{i,T-1} - y_{i,1}\right) + y_{i,T}\left(y_{i,T-2} - y_{i,0}\right)\right] = \frac{\sigma_\varepsilon^2}{1-\rho^2}\left(\rho^2 - \rho^T + 1 - \rho^{T-2}\right) \\
&= \frac{\sigma_\varepsilon^2}{1-\rho^2}\left(1 - \rho^{T-2}\right)\left(1 + \rho^2\right), \\
C_{n,T} &\xrightarrow{p} \mathrm{E}\left[y_{i,T}\left(y_{i,T-1} - y_{i,1}\right)\right] = \frac{\sigma_\varepsilon^2}{1-\rho^2}\left(\rho - \rho^{T-1}\right) = \frac{\rho\sigma_\varepsilon^2}{1-\rho^2}\left(1 - \rho^{T-2}\right).
\end{aligned}$$

As a result, our proposed estimator based on Equation (1) is given by

$$\widehat{\rho} = \frac{-B_{n,T}}{2A_{n,T}} \pm \sqrt{\left(\frac{-B_{n,T}}{2A_{n,T}}\right)^2 - \frac{C_{n,T}}{A_{n,T}}} \tag{4}$$

$$\begin{aligned}
&\xrightarrow{p} \frac{1+\rho^2}{2\rho} \pm \sqrt{\left(\frac{1+\rho^2}{2\rho}\right)^2 - 1} \\
&= \frac{1+\rho^2}{2\rho} \pm \sqrt{\frac{1+2\rho^2+\rho^4}{4\rho^2} - 1} \\
&= \frac{1+\rho^2}{2\rho} \pm \sqrt{\frac{1-2\rho^2+\rho^4}{4\rho^2}} \\
&= \frac{1+\rho^2}{2\rho} \pm \left|\frac{1-\rho^2}{2\rho}\right|.
\end{aligned} \tag{5}$$

The preceding result shows that our proposed estimator converges to two distinct real roots: the true value ρ and the reciprocal of the true value $1/\rho$. This result also holds when $T \to \infty$. As a consequence, one of the roots is consistent but determining which root is consistent depends on the sign of the true value of ρ.

2.2.2 The case when $\rho = 1$

A similar argument allows us to write the large-n limits of the coefficients of Equation (3) as

$$\begin{aligned}
A_{n,T} &\xrightarrow{p} \sigma_\varepsilon^2 (T-2), \\
-B_{n,T} &\xrightarrow{p} 2\sigma_\varepsilon^2 (T-2), \\
C_{n,T} &\xrightarrow{p} \sigma_\varepsilon^2 (T-2).
\end{aligned}$$

As a consequence of these large-n limits, Equation (3) will have only one real root with multiplicity 2. A similar argument as in the case where $|\rho| < 1$ allows us to conclude that

$$\widehat{\rho} = -\frac{B_{n,T}}{2A_{n,T}}$$

is the unique consistent root for the unit root case. This result also holds when $T \to \infty$.

2.2.3 Choosing the consistent root in practice

Following our preceding discussion, observe that the two roots of the quadratic Equation (3) will always have the same sign and will never be complex conjugates of each other. In addition, the two roots will also have the same sign as the true value ρ. A feasible selection rule to determine the consistent root is to choose the smaller of the two roots when both roots are positive and to choose the larger of the two roots when both roots are negative.

Unfortunately, this selection rule will not work all the time because the expression inside the square root (i.e., the discriminant) in Equation (4) may be negative. This is more likely to happen in two situations: (a) when the law of large numbers used to justify Equation (5) is a poor approximation of the finite sample behavior and (b) when $\rho \to 1$.

To avoid the possibility of roots being complex conjugates of each other, we exploit the result in Equation (5) along with the idempotence of the absolute value by calculating

$$\hat{\rho} = \frac{-B_{n,T}}{2A_{n,T}} \pm \sqrt{\left|\left(\frac{-B_{n,T}}{2A_{n,T}}\right)^2 - \frac{C_{n,T}}{A_{n,T}}\right|}$$

instead of Equation (4) and then choosing the larger or the smaller of the roots according to the selection rule.

The possibility of roots being complex conjugates of each other need not be such a practical burden. It can be used informally as a way to determine whether $\rho = 1$ or $|\rho| < 1$.

2.3 Asymptotic distribution of the consistent root

2.3.1 The case when $\rho = 1$

We start with the unit root case because we only have one consistent root given our preceding discussion. We can rewrite $-B_{n,T}$ as

$$-B_{n,T} = \frac{2}{n}\sum_{i=1}^{n} y_{i,T-1}\left(y_{i,T-2} - y_{i,0}\right) + \frac{1}{n}\sum_{i=1}^{n}\left[y_{i,T-1}\left(\varepsilon_{i,T-1} - \varepsilon_{i,1}\right) + \varepsilon_{i,T}\left(y_{i,T-2} - y_{i,0}\right)\right]. \tag{6}$$

This implies that

$$\widehat{\rho} - 1 = \frac{\frac{1}{n}\sum_{i=1}^{n}\left[y_{i,T-1}\left(\varepsilon_{i,T-1} - \varepsilon_{i,1}\right) + \varepsilon_{i,T}\left(y_{i,T-2} - y_{i,0}\right)\right]}{\frac{2}{n}\sum_{i=1}^{n} y_{i,T-1}\left(y_{i,T-2} - y_{i,0}\right)}. \tag{7}$$

The numerator has zero expectation and is an average of independent random variables with finite variance because of Assumptions A1 to A3. As a consequence, a central limit theorem applies to the numerator so that along with the Slutzky lemma, we have a fixed-T asymptotic result where

$$\sqrt{n}(\widehat{\rho} - 1) \xrightarrow{d} N\left(0, V\right). \tag{8}$$

The asymptotic variance can be estimated consistently from observables but it is instructive to derive an expression for the asymptotic variance that depends on the characteristics of the data generating process.

To begin with, take note that in the unit root case, we have

$$y_{i,t} = \alpha_i + u_{i,-1} + \sum_{s=0}^{t} \varepsilon_{i,s}.$$

We start by calculating the variance of the numerator in Equation (7). Note that

$$\operatorname{Var}\left[y_{i,T-1}\left(\varepsilon_{i,T-1} - \varepsilon_{i,1}\right) + \varepsilon_{i,T}\left(y_{i,T-2} - y_{i,0}\right)\right],$$

which can alternatively be expressed as

$$\begin{aligned} &\operatorname{Var}\left[y_{i,T-1}\left(\varepsilon_{i,T-1} - \varepsilon_{i,1}\right)\right] + \operatorname{Var}\left[\varepsilon_{i,T}\left(y_{i,T-2} - y_{i,0}\right)\right] + \\ &2\operatorname{Cov}\left[y_{i,T-1}\left(\varepsilon_{i,T-1} - \varepsilon_{i,1}\right), \varepsilon_{i,T}\left(y_{i,T-2} - y_{i,0}\right)\right]. \end{aligned} \tag{9}$$

For the first variance term in Equation (9), we have

$$\begin{aligned} &\operatorname{Var}\left[y_{i,T-1}\left(\varepsilon_{i,T-1} - \varepsilon_{i,1}\right)\right] \\ =&\operatorname{E}\left[y_{i,T-1}^2\left(\varepsilon_{i,T-1} - \varepsilon_{i,1}\right)^2\right] - \left(\operatorname{E}\left[y_{i,T-1}\left(\varepsilon_{i,T-1} - \varepsilon_{i,1}\right)\right]\right)^2 \\ =&\operatorname{E}\left[\left(\alpha_i + u_{i,-1} + \sum_{s=0}^{T-1}\varepsilon_{i,s}\right)^2\left(\varepsilon_{i,T-1} - \varepsilon_{i,1}\right)^2\right] - \left(\sigma_\varepsilon^2 - \sigma_\varepsilon^2\right)^2 \\ =&\operatorname{E}\left[\alpha_i^2\left(\varepsilon_{i,T-1} - \varepsilon_{i,1}\right)^2\right] \\ &+ \operatorname{E}\left[u_{i,-1}^2\left(\varepsilon_{i,T-1} - \varepsilon_{i,1}\right)^2\right] + \operatorname{E}\left[\left(\sum_{s=0}^{T-1}\varepsilon_{i,s}\right)^2\left(\varepsilon_{i,T-1}^2 - 2\varepsilon_{i,T-1}\varepsilon_{i,1} + \varepsilon_{i,1}^2\right)\right] \\ &+ 2\operatorname{E}\left[\alpha_i u_{i,-1}\left(\varepsilon_{i,T-1} - \varepsilon_{i,1}\right)^2\right] + 2\operatorname{E}\left[\alpha_i\left(\sum_{s=0}^{T-1}\varepsilon_{i,s}\right)\left(\varepsilon_{i,T-1} - \varepsilon_{i,1}\right)^2\right] \\ &+ 2\operatorname{E}\left[u_{i,-1}\left(\sum_{s=0}^{T-1}\varepsilon_{i,s}\right)\left(\varepsilon_{i,T-1} - \varepsilon_{i,1}\right)^2\right] \end{aligned}$$

$$
\begin{aligned}
&= \mathrm{E}\left(\alpha_i^2\right) \mathrm{E}\left[\left(\varepsilon_{i,T-1} - \varepsilon_{i,1}\right)^2\right] + \mathrm{E}\left(u_{i,-1}^2\right) \mathrm{E}\left[\left(\varepsilon_{i,T-1} - \varepsilon_{i,1}\right)^2\right] \\
&\quad + \sum_{s=0}^{T-1} \mathrm{E}\left[\varepsilon_{i,s}^2 \left(\varepsilon_{i,T-1}^2 - 2\varepsilon_{i,T-1}\varepsilon_{i,1} + \varepsilon_{i,1}^2\right)\right] \\
&\quad + 2 \sum_{s=0, r\neq s}^{T-1} \mathrm{E}\left[\varepsilon_{i,s}\varepsilon_{i,r} \left(\varepsilon_{i,T-1}^2 - 2\varepsilon_{i,T-1}\varepsilon_{i,1} + \varepsilon_{i,1}^2\right)\right] + 2\,\mathrm{E}\left(\alpha_i\right) \mathrm{E}\left(u_{i,-1}\right) \mathrm{E}\left[\left(\varepsilon_{i,T-1} - \varepsilon_{i,1}\right)^2\right] \\
&\quad + 2\,\mathrm{E}\left(\alpha_i\right) \mathrm{E}\left[\left(\sum_{s=0}^{T-1} \varepsilon_{i,s}\right) \left(\varepsilon_{i,T-1} - \varepsilon_{i,1}\right)^2\right] + 2\,\mathrm{E}\left(u_{i,-1}\right) \mathrm{E}\left[\left(\sum_{s=0}^{T-1} \varepsilon_{i,s}\right) \left(\varepsilon_{i,T-1} - \varepsilon_{i,1}\right)^2\right] \\
&= \left[\mathrm{E}\left(\alpha_i^2\right) + \mathrm{E}\left(u_{i,-1}^2\right)\right] \left(2\sigma_\varepsilon^2\right) + \sum_{s=0}^{T-1} \mathrm{E}\left(\varepsilon_{i,s}^2 \varepsilon_{i,T-1}^2\right) + \sum_{s=0}^{T-1} \mathrm{E}\left(\varepsilon_{i,s}^2 \varepsilon_{i,1}^2\right) \\
&\quad - 2 \sum_{s=0}^{T-1} \mathrm{E}\left[\varepsilon_{i,s}^2 \varepsilon_{i,T-1}\varepsilon_{i,1}\right] \\
&\quad - 4\,\mathrm{E}\left(\varepsilon_{i,T-1}^2 \varepsilon_{i,1}^2\right) + 2\,\mathrm{E}\left(\alpha_i\right) \mathrm{E}\left(\varepsilon_{i,1}^3 + \varepsilon_{i,T-1}^3\right) \\
&= \left[\mathrm{E}\left(\alpha_i^2\right) + \mathrm{E}\left(u_{i,-1}^2\right)\right] \left(2\sigma_\varepsilon^2\right) + (T-1)\,\sigma_\varepsilon^4 + \mathrm{E}\left(\varepsilon_{i,T-1}^4\right) + (T-1)\,\sigma_\varepsilon^4 + \mathrm{E}\left(\varepsilon_{i,1}^4\right) - 4\sigma_\varepsilon^4 \\
&\quad + 2\,\mathrm{E}\left(\alpha_i\right) \mathrm{E}\left(\varepsilon_{i,1}^3 + \varepsilon_{i,T-1}^3\right) \\
&= \left[\mathrm{E}\left(\alpha_i^2\right) + \mathrm{E}\left(u_{i,-1}^2\right)\right] \left(2\sigma_\varepsilon^2\right) + (2T-6)\,\sigma_\varepsilon^4 \\
&\quad + 2\,\mathrm{E}\left(\alpha_i\right) \mathrm{E}\left(\varepsilon_{i,1}^3 + \varepsilon_{i,T-1}^3\right) + \mathrm{E}\left(\varepsilon_{i,1}^4\right) + \mathrm{E}\left(\varepsilon_{i,T-1}^4\right).
\end{aligned}
$$

In the previous derivations, we simplify terms of the form $\mathrm{E}\left(\varepsilon_{i,s}^2 \varepsilon_{i,T-1}\varepsilon_{i,1}\right)$, $\mathrm{E}\left(\varepsilon_{i,s}^2 \varepsilon_{i,p}^2\right)$, $\mathrm{E}\left(\varepsilon_{i,s}\varepsilon_{i,r}\varepsilon_{i,p}^2\right)$, $\mathrm{E}\left(\varepsilon_{i,s}\varepsilon_{i,r}\varepsilon_{i,T-1}\varepsilon_{i,1}\right)$, $\mathrm{E}\left(\varepsilon_{i,s}\varepsilon_{i,T-1}\varepsilon_{i,1}\right)$, and $\mathrm{E}\left(\varepsilon_{i,s}\varepsilon_{i,p}^2\right)$ where $p \in \{1, T-1\}$, $r \neq s$, and $s = 0, \ldots, T-1$. Assumption A3 allows us to conclude that:

- $\mathrm{E}\left(\varepsilon_{i,s}^2 \varepsilon_{i,T-1}\varepsilon_{i,1}\right) = \mathrm{E}\left(\varepsilon_{i,T-1}^3 \varepsilon_{i,1}\right) = \mathrm{E}\left(\varepsilon_{i,1}^3 \varepsilon_{i,T-1}\right) = 0$
  and $\mathrm{E}\left(\varepsilon_{i,s}^2 \varepsilon_{i,T-1}\varepsilon_{i,1}\right) = \mathrm{E}\left(\varepsilon_{i,s}^2\right) \mathrm{E}\left(\varepsilon_{i,T-1}\varepsilon_{i,1}\right) = 0$ for $s \notin \{1, T-1\}$.
- $\mathrm{E}\left(\varepsilon_{i,s}^2 \varepsilon_{i,p}^2\right) = \mathrm{E}\left(\varepsilon_{i,p}^4\right)$ for $p = s$ and $\mathrm{E}\left(\varepsilon_{i,s}^2 \varepsilon_{i,p}^2\right) = \mathrm{E}\left(\varepsilon_{i,s}^2\right) \mathrm{E}\left(\varepsilon_{i,p}^2\right) = \sigma_\varepsilon^4$ for $p \neq s$.
- $\mathrm{E}\left(\varepsilon_{i,s}\varepsilon_{i,r}\varepsilon_{i,p}^2\right) = \mathrm{E}\left(\varepsilon_{i,r}\right) \mathrm{E}\left(\varepsilon_{i,s}\varepsilon_{i,p}^2\right) = 0$ for $r \neq s$ and $s = p$.
  Similarly, $\mathrm{E}\left(\varepsilon_{i,s}\varepsilon_{i,r}\varepsilon_{i,p}^2\right) = \mathrm{E}\left(\varepsilon_{i,s}\right) \mathrm{E}\left(\varepsilon_{i,r}\varepsilon_{i,p}^2\right) = 0$ for $r \neq s$ and $r = p$.
- $\mathrm{E}\left(\varepsilon_{i,s}\varepsilon_{i,r}\varepsilon_{i,T-1}\varepsilon_{i,1}\right) = 0$ whenever $s, r \notin \{1, T-1\}$.
  Otherwise, $\mathrm{E}\left(\varepsilon_{i,s}\varepsilon_{i,r}\varepsilon_{i,T-1}\varepsilon_{i,1}\right) = \mathrm{E}\left(\varepsilon_{i,T-1}^2 \varepsilon_{i,1}^2\right) = \sigma_\varepsilon^4$.
- $\mathrm{E}\left(\varepsilon_{i,s}\varepsilon_{i,T-1}\varepsilon_{i,1}\right) = \mathrm{E}\left(\varepsilon_{i,s}\right) \mathrm{E}\left(\varepsilon_{i,T-1}\varepsilon_{i,1}\right) = 0$ whenever $s \notin \{1, T-1\}$.
  Otherwise, $\mathrm{E}\left(\varepsilon_{i,s}\varepsilon_{i,T-1}\varepsilon_{i,1}\right) = \mathrm{E}\left(\varepsilon_{i,1}\right) \mathrm{E}\left(\varepsilon_{i,T-1}\varepsilon_{i,s}\right) = 0$.
- $\mathrm{E}\left(\varepsilon_{i,s}\varepsilon_{i,p}^2\right) = \mathrm{E}\left(\varepsilon_{i,s}\right) \mathrm{E}\left(\varepsilon_{i,p}^2\right) = 0$ whenever $s \neq p$.
  Otherwise, $\mathrm{E}\left(\varepsilon_{i,s}\varepsilon_{i,p}^2\right) = \mathrm{E}\left(\varepsilon_{i,p}^3\right)$.

Note that if we assume that $\mathrm{E}(u_{i,-1}) \neq 0$, the first variance term in Equation (9) is

$$\begin{aligned}
&\mathrm{Var}\left[y_{i,T-1}\left(\varepsilon_{i,T-1}-\varepsilon_{i,1}\right)\right] \\
=&\left[\mathrm{E}\left(\alpha_i^2\right)+2\,\mathrm{E}\left(\alpha_i\right)\mathrm{E}\left(u_{i,-1}\right)+\mathrm{E}\left(u_{i,-1}^2\right)\right]\left(2\sigma_\varepsilon^2\right)+\left(2T-6\right)\sigma_\varepsilon^4 \\
&+2\left[\mathrm{E}\left(\alpha_i\right)+\mathrm{E}\left(u_{i,-1}\right)\right]\mathrm{E}\left(\varepsilon_{i,1}^3+\varepsilon_{i,T-1}^3\right)+\mathrm{E}\left(\varepsilon_{i,1}^4\right)+\mathrm{E}\left(\varepsilon_{i,T-1}^4\right).
\end{aligned}$$

For the second variance term in Equation (9), the following expression results

$$\begin{aligned}
&Var\left[\varepsilon_{i,T}\left(y_{i,T-2}-y_{i,0}\right)\right] \\
&=\mathrm{E}\left[\varepsilon_{i,T}^2\left(y_{i,T-2}-y_{i,0}\right)^2\right]-\left(\mathrm{E}\left[\varepsilon_{i,T}\left(y_{i,T-2}-y_{i,0}\right)\right]\right)^2 \\
&=\mathrm{E}\left[\varepsilon_{i,T}^2\left(\sum_{s=1}^{T-2}\varepsilon_{i,s}\right)^2\right]-\left(\mathrm{E}\left[\varepsilon_{i,T}\left(\sum_{s=1}^{T-2}\varepsilon_{i,s}\right)\right]\right)^2 \\
&=\mathrm{E}\left[\varepsilon_{i,T}^2\left(\sum_{s=1}^{T-2}\varepsilon_{i,s}^2\right)\right]+\mathrm{E}\left[2\varepsilon_{i,T}^2\left(\sum_{s=1,r\neq s}^{T-2}\varepsilon_{i,s}\varepsilon_{i,r}\right)\right]+0 \\
&=\sum_{s=1}^{T-2}\sigma_\varepsilon^2\left(\sigma_\varepsilon^2\right) \\
&=\left(T-2\right)\sigma_\varepsilon^4.
\end{aligned}$$

Finally, the covariance term in Equation (9) reduces to zero as follows:

$$\begin{aligned}
&\mathrm{Cov}\left[y_{i,T-1}\left(\varepsilon_{i,T-1}-\varepsilon_{i,1}\right),\varepsilon_{i,T}\left(y_{i,T-2}-y_{i,0}\right)\right] \\
=&\,\mathrm{E}\left[y_{i,T-1}\left(\varepsilon_{i,T-1}-\varepsilon_{i,1}\right)\varepsilon_{i,T}\left(y_{i,T-2}-y_{i,0}\right)\right] \\
&-\mathrm{E}\left[y_{i,T-1}\left(\varepsilon_{i,T-1}-\varepsilon_{i,1}\right)\right]\mathrm{E}\left[\varepsilon_{i,T}\left(y_{i,T-2}-y_{i,0}\right)\right] \\
=&\,\mathrm{E}\left[\left(\alpha_i+u_{i,-1}+\sum_{s=0}^{T-1}\varepsilon_{i,s}\right)\left(\varepsilon_{i,T-1}-\varepsilon_{i,1}\right)\varepsilon_{i,T}\left(\sum_{s=1}^{T-2}\varepsilon_{i,s}\right)\right] \\
&-\mathrm{E}\left[\left(\alpha_i+u_{i,-1}+\sum_{s=0}^{T-1}\varepsilon_{i,s}\right)\left(\varepsilon_{i,T-1}-\varepsilon_{i,1}\right)\right]\mathrm{E}\left[\varepsilon_{i,T}\left(\sum_{s=1}^{T-2}\varepsilon_{i,s}\right)\right] \\
=&\,\mathrm{E}\left[\left(\sum_{s=0}^{T-1}\varepsilon_{i,s}\right)\left(\varepsilon_{i,T-1}-\varepsilon_{i,1}\right)\varepsilon_{i,T}\left(\sum_{s=1}^{T-2}\varepsilon_{i,s}\right)\right]-0 \\
=&\,\mathrm{E}\left[\left(\sum_{s=0}^{T-1}\varepsilon_{i,s}\right)\varepsilon_{i,T-1}\varepsilon_{i,T}\left(\sum_{s=1}^{T-2}\varepsilon_{i,s}\right)\right]-\mathrm{E}\left[\left(\sum_{s=0}^{T-1}\varepsilon_{i,s}\right)\varepsilon_{i,1}\varepsilon_{i,T}\left(\sum_{s=1}^{T-2}\varepsilon_{i,s}\right)\right] \\
=&\,\mathrm{E}\left[\left(\varepsilon_{i,T-1}+\sum_{s=0}^{T-2}\varepsilon_{i,s}\right)\varepsilon_{i,T-1}\varepsilon_{i,T}\left(\sum_{s=1}^{T-2}\varepsilon_{i,s}\right)\right] \\
&-\mathrm{E}\left[\left(\varepsilon_{i,0}+\varepsilon_{i,1}+\sum_{s=2}^{T-2}\varepsilon_{i,s}+\varepsilon_{i,T-1}\right)\varepsilon_{i,1}\varepsilon_{i,T}\left(\varepsilon_{i,1}+\sum_{s=2}^{T-2}\varepsilon_{i,s}\right)\right]
\end{aligned}$$

$$
\begin{aligned}
&= \mathrm{E}\left[\varepsilon_{i,T-1}^{2}\varepsilon_{i,T}\left(\sum_{s=1}^{T-2}\varepsilon_{i,s}\right)\right] + \mathrm{E}\left[\left(\sum_{s=0}^{T-2}\varepsilon_{i,s}\right)^{2}\varepsilon_{i,T-1}\varepsilon_{i,T}\right] - \mathrm{E}\left[\varepsilon_{i,0}\varepsilon_{i,1}\varepsilon_{i,T}\left(\varepsilon_{i,1}+\sum_{s=2}^{T-2}\varepsilon_{i,s}\right)\right] \\
&\quad - \mathrm{E}\left[\varepsilon_{i,1}\varepsilon_{i,T}\left(\varepsilon_{i,1}+\sum_{s=2}^{T-2}\varepsilon_{i,s}\right)^{2}\right] - \mathrm{E}\left[\varepsilon_{i,T-1}\varepsilon_{i,1}\varepsilon_{i,T}\left(\varepsilon_{i,1}+\sum_{s=2}^{T-2}\varepsilon_{i,s}\right)\right] \\
&= \mathrm{E}\left(\varepsilon_{i,T-1}^{2}\right)\mathrm{E}\left(\varepsilon_{i,T}\right)\mathrm{E}\left[\left(\sum_{s=1}^{T-2}\varepsilon_{i,s}\right)\right] + \mathrm{E}\left[\left(\sum_{s=0}^{T-2}\varepsilon_{i,s}\right)^{2}\right]\mathrm{E}\left(\varepsilon_{i,T-1}\varepsilon_{i,T}\right) \\
&\quad - \mathrm{E}\left[\varepsilon_{i,0}\varepsilon_{i,1}\left(\varepsilon_{i,1}+\sum_{s=2}^{T-2}\varepsilon_{i,s}\right)\right]\mathrm{E}\left(\varepsilon_{i,T}\right) - \mathrm{E}\left[\varepsilon_{i,1}\left(\varepsilon_{i,1}+\sum_{s=2}^{T-2}\varepsilon_{i,s}\right)^{2}\right]\mathrm{E}\left(\varepsilon_{i,T}\right) \\
&\quad - \mathrm{E}\left[\varepsilon_{i,1}\left(\varepsilon_{i,1}+\sum_{s=2}^{T-2}\varepsilon_{i,s}\right)\right]\mathrm{E}\left(\varepsilon_{i,T-1}\varepsilon_{i,T}\right) \\
&= 0.
\end{aligned}
$$

Using $H_{n,T}$ to represent the numerator of Equation (7) and collecting the three results on the terms in Equation (9) gives

$$
\sqrt{n}H_{n,T} \xrightarrow{d} N\left(0, V_H\right),
$$

where

$$
V_H = 2\sigma_{\varepsilon}^{2}\left(\mathrm{E}\left(\alpha_i^2\right) + \mathrm{E}\left(u_{i,-1}^{2}\right)\right) + \left(3T-8\right)\sigma_{\varepsilon}^{4} + 4\,\mathrm{E}\left(\alpha_i\right)\mathrm{E}\left(\varepsilon_{i,1}^{3}\right) + 2\,\mathrm{E}\left(\varepsilon_{i,1}^{4}\right).
$$

Since $A_{n,T} \xrightarrow{p} \sigma_{\varepsilon}^{2}\left(T-2\right)$, we have the result given in Equation (8) and the corresponding asymptotic variance V results from an application of Slutzky's lemma:[5]

$$
V = \frac{\mathrm{E}\left(\alpha_i^2\right) + \mathrm{E}\left(u_{i,-1}^{2}\right)}{2\sigma_{\varepsilon}^{2}\left(T-2\right)^{2}} + \frac{3T-8}{4\left(T-2\right)^{2}} + \frac{\mathrm{E}\left(\varepsilon_{i,1}^{4}\right)}{2\sigma_{\varepsilon}^{4}\left(T-2\right)^{2}} + \frac{\mathrm{E}\left(\alpha_i\right)\mathrm{E}\left(\varepsilon_{i,1}^{3}\right)}{\sigma_{\varepsilon}^{4}\left(T-2\right)^{2}}. \tag{10}
$$

Under Assumption A4, we can now derive the large-n, large-T distribution of $\widehat{\rho}$ as

$$
\sqrt{nT}(\widehat{\rho}-1) \xrightarrow{d} N\left(0, \frac{3}{4}\right). \tag{11}
$$

The practical relevance of these results stem from the ways in which the characteristics of the idiosyncratic remainder component affects inferences based on the asymptotic approximation provided by the large-n distribution in Equation (8). In particular, the fourth moment of the idiosyncratic remainder component matters for inference but its influence disappears when $T \to \infty$. Similarly, a nonzero mean for the individual-specific fixed effect α_i interacts with the third moment of the idiosyncratic remainder component. This latter result implies that assuming zero mean for α_i is with loss of generality. However, these nuisance parameters which are specific to the data generating process will not matter when $T \to \infty$.

[5]Note that it is easy to modify the result to allow for the possibility that $\mathrm{E}\left(u_{i,-1}\right) \neq 0$.

2.3.2 The case when $|\rho| < 1$

We now consider the stationary case and assume that $\rho > 0$. By our analysis of consistency, we know that the smaller root

$$\widehat{\rho} = \frac{-B_{n,T}}{2A_{n,T}} - \sqrt{\left(\frac{-B_{n,T}}{2A_{n,T}}\right)^2 - \frac{C_{n,T}}{A_{n,T}}}$$

is the consistent root. To derive the asymptotic distribution, we have to take care of the asymptotic behavior of the term inside the square root. To facilitate the analysis, we write

$$\begin{aligned}\frac{-B_{n,T}}{2A_{n,T}} &= \rho + \frac{1}{2}H_1 + \frac{1}{2}H_2, \\ \frac{C_{n,T}}{A_{n,T}} &= \rho^2 + \rho H_1 + \rho H_2 + H_3,\end{aligned}$$

where

$$\begin{aligned}H_1 &= \frac{\frac{1}{n}\sum_{i=1}^{n} y_{i,T-1}\left(\varepsilon_{i,T-1} - \varepsilon_{i,1}\right)}{\frac{1}{n}\sum_{i=1}^{n} y_{i,T-1}\left(y_{i,T-2} - y_{i,0}\right)}, \\ H_2 &= \frac{\frac{1}{n}\sum_{i=1}^{n}\left(\left(1-\rho\right)\alpha_i + \varepsilon_{i,T}\right)\left(y_{i,T-2} - y_{i,0}\right)}{\frac{1}{n}\sum_{i=1}^{n} y_{i,T-1}\left(y_{i,T-2} - y_{i,0}\right)}, \\ H_3 &= \frac{\frac{1}{n}\sum_{i=1}^{n}\left(\left(1-\rho\right)\alpha_i + \varepsilon_{i,T}\right)\left(\varepsilon_{i,T-1} - \varepsilon_{i,1}\right)}{\frac{1}{n}\sum_{i=1}^{n} y_{i,T-1}\left(y_{i,T-2} - y_{i,0}\right)}.\end{aligned}$$

As a consequence, we have

$$\begin{aligned}\widehat{\rho} &= \rho + \frac{1}{2}H_1 + \frac{1}{2}H_2 - \sqrt{\left(\rho + \frac{1}{2}H_1 + \frac{1}{2}H_2\right)^2 - \left(\rho^2 + \rho H_1 + \rho H_2 + H_3\right)} \\ &= \rho + \frac{1}{2}H_1 + \frac{1}{2}H_2 - \sqrt{\left(\frac{1}{2}H_1 + \frac{1}{2}H_2\right)^2 - H_3}.\end{aligned}$$

If we let

$$H = H_1 + H_2 - \mathrm{E}\left(H_1 + H_2\right),$$

then the representation for $\widehat{\rho}$ can now be expressed as

$$\begin{aligned}
\widehat{\rho} &= \rho + \frac{1}{2}H + \frac{1}{2}\,\mathrm{E}\left(H_1 + H_2\right) - \sqrt{\left(\frac{1}{2}H\right)^2 + \frac{1}{2}H + \frac{1}{4}\left[\mathrm{E}\left(H_1 + H_2\right)\right]^2 - H_3} \\
&= \rho + \frac{1}{2}H + \frac{1}{2}\,\mathrm{E}\left(H_1 + H_2\right) - \sqrt{O_p\left(n^{-1}\right) + O_p\left(n^{-1/2}\right) + O_p\left(1\right) - O_p\left(n^{-1/2}\right)} \\
&= \rho + \frac{1}{2}H + \frac{1}{2}\,\mathrm{E}\left(H_1 + H_2\right) - \left|\frac{1}{2}\,\mathrm{E}\left(H_1 + H_2\right)\right| \\
&= \rho + \frac{1}{2}H. \qquad (12)
\end{aligned}$$

The second step in the derivation makes use of the following observations: (a) a central limit theorem applies to the zero-mean random variables H and H_3 and (b) the mean $\mathrm{E}\left(H_1 + H_2\right)$ is a bounded constant. The third step of the derivation indicates that the $O_p\left(1\right)$ term dominates the other terms inside the square root. The final step arises because the mean $\mathrm{E}\left(H_1 + H_2\right)$ is positive whenever $|\rho| < 1$. The result in Equation (12) implies that the asymptotic distribution of $\widehat{\rho}$ follows the asymptotic distribution of H.

In order to derive the fixed-T and large-T distribution of $\widehat{\rho}$, we start by calculating the asymptotic variance of the numerator of $H_1 + H_2$. Note that

$$\mathrm{Var}\left[y_{i,T-1}\left(\varepsilon_{i,T-1} - \varepsilon_{i,1}\right) + \left(\left(1-\rho\right)\alpha_i + \varepsilon_{i,T}\right)\left(y_{i,T-2} - y_{i,0}\right)\right],$$

which can alternatively be represented by the expression

$$\begin{aligned}
&\mathrm{Var}\left[y_{i,T-1}\left(\varepsilon_{i,T-1} - \varepsilon_{i,1}\right)\right] + \mathrm{Var}\left[\left(\left(1-\rho\right)\alpha_i + \varepsilon_{i,T}\right)\left(y_{i,T-2} - y_{i,0}\right)\right] \qquad (13) \\
&+ 2\,\mathrm{Cov}\left[y_{i,T-1}\left(\varepsilon_{i,T-1} - \varepsilon_{i,1}\right), \left(\left(1-\rho\right)\alpha_i + \varepsilon_{i,T}\right)\left(y_{i,T-2} - y_{i,0}\right)\right].
\end{aligned}$$

Furthermore, we have the following expression for $u_{i,t}$:

$$u_{i,t} = \rho^{t+1}u_{i,-1} + \sum_{s=0}^{t}\rho^{t-s}\varepsilon_{i,s}.$$

The first variance in Equation (13) can also be expressed as

$$\begin{aligned}
&\mathrm{Var}\left[y_{i,T-1}\left(\varepsilon_{i,T-1} - \varepsilon_{i,1}\right)\right] \\
&= \mathrm{Var}\left[\left(\alpha_i + u_{i,T-1}\right)\left(\varepsilon_{i,T-1} - \varepsilon_{i,1}\right)\right] \\
&= \mathrm{E}\left[\left(\alpha_i + u_{i,T-1}\right)^2\left(\varepsilon_{i,T-1} - \varepsilon_{i,1}\right)^2\right] - \left(\mathrm{E}\left[\left(\alpha_i + u_{i,T-1}\right)\left(\varepsilon_{i,T-1} - \varepsilon_{i,1}\right)\right]\right)^2
\end{aligned}$$

$$
\begin{aligned}
=&\mathrm{E}\left[\alpha_i^2\left(\varepsilon_{i,T-1}-\varepsilon_{i,1}\right)^2\right]+2\,\mathrm{E}\left[\alpha_i u_{i,T-1}\left(\varepsilon_{i,T-1}-\varepsilon_{i,1}\right)^2\right]+\mathrm{E}\left[u_{i,T-1}^2\left(\varepsilon_{i,T-1}-\varepsilon_{i,1}\right)^2\right]\\
&-\left(\mathrm{E}\left[\alpha_i\left(\varepsilon_{i,T-1}-\varepsilon_{i,1}\right)\right]+\mathrm{E}\left[u_{i,T-1}\left(\varepsilon_{i,T-1}-\varepsilon_{i,1}\right)\right]\right)^2\\
=&2\sigma_\varepsilon^2\,\mathrm{E}\left(\alpha_i^2\right)+2\,\mathrm{E}\left(\alpha_i\right)\mathrm{E}\left[u_{i,T-1}\left(\varepsilon_{i,T-1}-\varepsilon_{i,1}\right)^2\right]+\mathrm{E}\left[u_{i,T-1}^2\left(\varepsilon_{i,T-1}-\varepsilon_{i,1}\right)^2\right]\\
&-\left(\mathrm{E}\left[u_{i,T-1}\left(\varepsilon_{i,T-1}-\varepsilon_{i,1}\right)\right]\right)^2\\
=&2\sigma_\varepsilon^2\,\mathrm{E}\left(\alpha_i^2\right)+2\,\mathrm{E}\left(\alpha_i\right)\mathrm{E}\left[\left(\rho^T u_{i,-1}+\sum_{s=0}^{T-1}\rho^{T-1-s}\varepsilon_{i,s}\right)\left(\varepsilon_{i,T-1}-\varepsilon_{i,1}\right)^2\right]\\
&+\mathrm{E}\left[\left(\rho^T u_{i,-1}+\sum_{s=0}^{T-1}\rho^{T-1-s}\varepsilon_{i,s}\right)^2\left(\varepsilon_{i,T-1}-\varepsilon_{i,1}\right)^2\right]\\
&-\left(\mathrm{E}\left[\left(\rho^T u_{i,-1}+\sum_{s=0}^{T-1}\rho^{T-1-s}\varepsilon_{i,s}\right)\left(\varepsilon_{i,T-1}-\varepsilon_{i,1}\right)\right]\right)^2\\
=&2\sigma_\varepsilon^2\,\mathrm{E}\left(\alpha_i^2\right)+2\,\mathrm{E}\left(\alpha_i\right)\mathrm{E}\left[\varepsilon_{i,T-1}^3+\rho^{T-2}\varepsilon_{i,1}^3\right]+\mathrm{E}\left(\rho^{2T}u_{i,-1}^2\right)\mathrm{E}\left[\left(\varepsilon_{i,T-1}-\varepsilon_{i,1}\right)^2\right]\\
&+\sum_{s=0}^{T-1}\rho^{2(T-1-s)}\,\mathrm{E}\left[\varepsilon_{i,s}^2\left(\varepsilon_{i,T-1}-\varepsilon_{i,1}\right)^2\right]+2\,\mathrm{E}\left[\left(\rho^{T-2}\varepsilon_{i,1}\varepsilon_{i,T-1}\right)\left(\varepsilon_{i,T-1}-\varepsilon_{i,1}\right)^2\right]\\
&-\left(\mathrm{E}\left[\left(\rho^{T-2}\varepsilon_{i,1}+\varepsilon_{i,T-1}\right)\left(\varepsilon_{i,T-1}-\varepsilon_{i,1}\right)\right]\right)^2\\
=&2\sigma_\varepsilon^2\,\mathrm{E}\left(\alpha_i^2\right)+2\left(1+\rho^{T-2}\right)\mathrm{E}\left(\alpha_i\right)\mathrm{E}\left(\varepsilon_{i,1}^3\right)+2\sigma_\varepsilon^2\rho^{2T}\,\mathrm{E}\left(u_{i,-1}^2\right)+\mathrm{E}\left[\varepsilon_{i,T-1}^2\left(\varepsilon_{i,T-1}-\varepsilon_{i1}\right)^2\right]\\
&+\rho^{2(T-1)}\,\mathrm{E}\left[\varepsilon_{i,0}^2\left(\varepsilon_{i,T-1}-\varepsilon_{i,1}\right)^2\right]+\rho^{2(T-2)}\,\mathrm{E}\left[\varepsilon_{i,1}^2\left(\varepsilon_{i,T-1}-\varepsilon_{i,1}\right)^2\right]\\
&+\sum_{s=2}^{T-2}\rho^{2(T-1-s)}\,\mathrm{E}\left[\varepsilon_{i,s}^2\left(\varepsilon_{i,T-1}-\varepsilon_{i,1}\right)^2\right]-4\rho^{T-2}\,\mathrm{E}\left(\varepsilon_{i,1}^2\varepsilon_{i,T-1}^2\right)-\left(\sigma_\varepsilon^2\left(1-\rho^{T-2}\right)\right)^2\\
=&2\sigma_\varepsilon^2\,\mathrm{E}\left(\alpha_i^2\right)+2\left(1+\rho^{T-2}\right)\mathrm{E}\left(\alpha_i\right)\mathrm{E}\left(\varepsilon_{i,1}^3\right)+2\sigma_\varepsilon^2\rho^{2T}\,\mathrm{E}\left(u_{i,-1}^2\right)+\mathrm{E}\left(\varepsilon_{i,T-1}^4\right)\\
&+\sigma_\varepsilon^4+\rho^{2(T-1)}\left(\sigma_\varepsilon^4+\sigma_\varepsilon^4\right)+\rho^{2(T-2)}\left[\sigma_\varepsilon^4+\mathrm{E}\left(\varepsilon_{i,1}^4\right)\right]\\
&+\sum_{s=2}^{T-2}\rho^{2(T-1-s)}\,\mathrm{E}\left(\varepsilon_{i,s}^2\right)\mathrm{E}\left[\left(\varepsilon_{i,T-1}-\varepsilon_{i,1}\right)^2\right]-4\rho^{T-2}\sigma_\varepsilon^4-\left(\sigma_\varepsilon^2\left(1-\rho^{T-2}\right)\right)^2\\
=&2\sigma_\varepsilon^2\,\mathrm{E}\left(\alpha_i^2\right)+2\left(1+\rho^{T-2}\right)\mathrm{E}\left(\alpha_i\right)\mathrm{E}\left(\varepsilon_{i,1}^3\right)+2\sigma_\varepsilon^2\rho^{2T}\,\mathrm{E}\left(u_{i,-1}^2\right)+\mathrm{E}\left(\varepsilon_{i,T-1}^4\right)\\
&+\sigma_\varepsilon^4+\rho^{2(T-1)}\left(\sigma_\varepsilon^4+\sigma_\varepsilon^4\right)+\rho^{2(T-2)}\left[\sigma_\varepsilon^4+\mathrm{E}\left(\varepsilon_{i,1}^4\right)\right]+2\sigma_\varepsilon^4\sum_{s=2}^{T-2}\rho^{2(T-1-s)}-4\rho^{T-2}\sigma_\varepsilon^4\\
&-\left(\sigma_\varepsilon^2\left(1-\rho^{T-2}\right)\right)^2\\
=&2\sigma_\varepsilon^2\,\mathrm{E}\left(\alpha_i^2\right)+2\left(1+\rho^{T-2}\right)\mathrm{E}\left(\alpha_i\right)\mathrm{E}\left(\varepsilon_{i,1}^3\right)+2\sigma_\varepsilon^2\rho^{2T}\,\mathrm{E}\left(u_{i,-1}^2\right)+\mathrm{E}\left(\varepsilon_{i,T-1}^4\right)\\
&+\sigma_\varepsilon^4+\rho^{2(T-1)}\left(\sigma_\varepsilon^4+\sigma_\varepsilon^4\right)+\rho^{2(T-2)}\left[\sigma_\varepsilon^4+\mathrm{E}\left(\varepsilon_{i,1}^4\right)\right]+2\sigma_\varepsilon^4\frac{\rho^4\left(1-\rho^{2(T-3)}\right)}{1-\rho^2}-4\rho^{T-2}\sigma_\varepsilon^4\\
&-\left(\sigma_\varepsilon^2\left(1-\rho^{T-2}\right)\right)^2.
\end{aligned}
$$

The second variance in Equation (13) can be expressed as

$$
\begin{aligned}
&\operatorname{Var}\left[\left((1-\rho)\alpha_i+\varepsilon_{i,T}\right)\left(y_{i,T-2}-y_{i,0}\right)\right]\\
=&\operatorname{Var}\left[\left((1-\rho)\alpha_i+\varepsilon_{i,T}\right)\left(u_{i,T-2}-u_{i,0}\right)\right]\\
=&\operatorname{E}\left[\left((1-\rho)\alpha_i+\varepsilon_{i,T}\right)^2\left(u_{i,T-2}-u_{i,0}\right)^2\right]-\left(\operatorname{E}\left[\left((1-\rho)\alpha_i+\varepsilon_{i,T}\right)\left(u_{i,T-2}-u_{i,0}\right)\right]\right)^2\\
=&\operatorname{E}\left[\left((1-\rho)\alpha_i+\varepsilon_{i,T}\right)^2\right]\operatorname{E}\left[\left(u_{i,T-2}-u_{i,0}\right)^2\right]-\left(\operatorname{E}\left((1-\rho)\alpha_i+\varepsilon_{i,T}\right)\operatorname{E}\left(u_{i,T-2}-u_{i,0}\right)\right)^2\\
=&\left[(1-\rho)^2\operatorname{E}\left(\alpha_i^2\right)+\sigma_\varepsilon^2\right]\operatorname{E}\left[\rho^2\left(u_{i,T-3}-u_{i,-1}\right)^2\right.\\
&\left.+2\rho\left(u_{i,T-3}-u_{i,-1}\right)\left(\varepsilon_{i,T-2}-\varepsilon_{i,0}\right)+\left(\varepsilon_{i,T-2}-\varepsilon_{i,0}\right)^2\right]-0\\
=&\left[(1-\rho)^2\operatorname{E}\left(\alpha_i^2\right)+\sigma_\varepsilon^2\right]\left[\operatorname{E}\left[\left(\rho^{T-2}-1\right)^2u_{i,-1}^2+\sum_{s=0}^{T-3}\rho^{2(T-3-s)}\varepsilon_{i,s}^2\right]-\right.\\
&\left.2\rho\operatorname{E}\left[u_{i,T-3}\varepsilon_{i,0}\right]+2\sigma_\varepsilon^2\right]\\
=&\left[(1-\rho)^2\operatorname{E}\left(\alpha_i^2\right)+\sigma_\varepsilon^2\right]\left[\left(1-\rho^{T-2}\right)^2\operatorname{E}\left(u_{i,-1}^2\right)+\sigma_\varepsilon^2\frac{\left(1-\rho^{2(T-2)}\right)}{1-\rho^2}-2\rho^{T-2}\sigma_\varepsilon^2+2\sigma_\varepsilon^2\right].
\end{aligned}
$$

The covariance term in Equation (13) can be expressed as

$$
\begin{aligned}
&\operatorname{Cov}\left[y_{i,T-1}\left(\varepsilon_{i,T-1}-\varepsilon_{i,1}\right),\left((1-\rho)\alpha_i+\varepsilon_{i,T}\right)\left(y_{i,T-2}-y_{i,0}\right)\right]\\
=&\operatorname{Cov}\left[\left(\alpha_i+u_{i,T-1}\right)\left(\varepsilon_{i,T-1}-\varepsilon_{i,1}\right),\left((1-\rho)\alpha_i+\varepsilon_{i,T}\right)\left(u_{i,T-2}-u_{i,0}\right)\right].\\
=&\operatorname{Cov}\left[\left(\alpha_i+u_{i,T-1}\right)\varepsilon_{i,T-1},(1-\rho)\alpha_iu_{i,T-2}\right]-\operatorname{Cov}\left[\left(\alpha_i+u_{i,T-1}\right)\varepsilon_{i,1},(1-\rho)\alpha_iu_{i,T-2}\right]\\
&-\operatorname{Cov}\left[\left(\alpha_i+u_{i,T-1}\right)\varepsilon_{i,T-1},(1-\rho)\alpha_iu_{i,0}\right]+\operatorname{Cov}\left[\left(\alpha_i+u_{i,T-1}\right)\varepsilon_{i,1},(1-\rho)\alpha_iu_{i,0}\right]\\
&+\operatorname{Cov}\left[\left(\alpha_i+u_{i,T-1}\right)\left(\varepsilon_{i,T-1}-\varepsilon_{i,1}\right),\varepsilon_{i,T}\left(u_{i,T-2}-u_{i,0}\right)\right]\\
=&0-(1-\rho)\left[\operatorname{E}\left(\alpha_i^2\right)\operatorname{E}\left(\varepsilon_{i,1}u_{i,T-2}\right)+\operatorname{E}\left(\alpha_i\right)\operatorname{E}\left(\varepsilon_{i,1}u_{i,T-2}u_{i,T-1}\right)\right]-0+0+0\\
=&-(1-\rho)\left[\operatorname{E}\left(\alpha_i^2\right)\rho^{T-3}\sigma_\varepsilon^2+\operatorname{E}\left(\alpha_i\right)\rho^{2T-5}\operatorname{E}\left(\varepsilon_{i,1}^3\right)\right].
\end{aligned}
$$

Note that the final step uses the following results:

- $\operatorname{Cov}\left[\left(\alpha_i+u_{i,T-1}\right)\varepsilon_{i,T-1},(1-\rho)\alpha_iu_{i,T-2}\right]=$
  $(1-\rho)\left[\operatorname{E}\left(\alpha_i^2\varepsilon_{i,T-1}u_{i,T-2}\right)+\operatorname{E}\left(u_{i,T-1}\varepsilon_{i,T-1}\alpha_iu_{i,T-2}\right)\right]=0.$
- $\operatorname{Cov}\left[\left(\alpha_i+u_{i,T-1}\right)\varepsilon_{i,1},(1-\rho)\alpha_iu_{i,T-2}\right]=$
  $(1-\rho)\left[\operatorname{E}\left(\alpha_i^2\right)\operatorname{E}\left(\varepsilon_{i,1}u_{i,T-2}\right)+\operatorname{E}\left(\alpha_i\right)\operatorname{E}\left(\varepsilon_{i,1}u_{i,T-2}u_{i,T-1}\right)\right].$
- $\operatorname{Cov}\left[\left(\alpha_i+u_{i,T-1}\right)\varepsilon_{i,T-1},(1-\rho)\alpha_iu_{i,0}\right]=$
  $(1-\rho)\left[\operatorname{E}\left(\alpha_i^2\varepsilon_{i,T-1}u_{i,0}\right)+\operatorname{E}\left(u_{i,T-1}\varepsilon_{i,T-1}\alpha_iu_{i,0}\right)\right]=0.$
- $\operatorname{Cov}\left[\left(\alpha_i+u_{i,T-1}\right)\varepsilon_{i,1},(1-\rho)\alpha_iu_{i,0}\right]=$
  $(1-\rho)\left[\operatorname{E}\left(\alpha_i^2\varepsilon_{i,1}u_{i,0}\right)+\operatorname{E}\left(u_{i,T-1}\varepsilon_{i,1}\alpha_iu_{i,0}\right)\right]=0.$

- $\mathrm{Cov}\left[\left(\alpha_i+u_{i,T-1}\right)\left(\varepsilon_{i,T-1}-\varepsilon_{i,1}\right),\varepsilon_{i,T}\left(u_{i,T-2}-u_{i,0}\right)\right]=0$ because of the presence of $\varepsilon_{i,T}$.

Collecting all the preceding results on the terms in Equation (13) gives us

$$\sqrt{n}H\xrightarrow{d}N\left(0,V_H\right),$$

where the corresponding asymptotic variance is

$$\begin{aligned}
V_H=&\,2\sigma_\varepsilon^2\,\mathrm{E}\left(\alpha_i^2\right)+2\left(1+\rho^{T-2}\right)\mathrm{E}\left(\alpha_i\right)\mathrm{E}\left(\varepsilon_{i,1}^3\right)+2\sigma_\varepsilon^2\rho^{2T}\,\mathrm{E}\left(u_{i,-1}^2\right)+\mathrm{E}\left(\varepsilon_{i,T-1}^4\right)\\
&+\sigma_\varepsilon^4+2\rho^{2(T-1)}\sigma_\varepsilon^4\\
&+\rho^{2(T-2)}\left[\sigma_\varepsilon^4+\mathrm{E}\left(\varepsilon_{i,1}^4\right)\right]+2\sigma_\varepsilon^4\frac{\rho^4\left(1-\rho^{2(T-3)}\right)}{1-\rho^2}-4\rho^{T-2}\sigma_\varepsilon^4-\left(\sigma_\varepsilon^2\left(1-\rho^{T-2}\right)\right)^2\\
&+\left[(1-\rho)^2\,\mathrm{E}\left(\alpha_i^2\right)+\sigma_\varepsilon^2\right]\left[\left(1-\rho^{T-2}\right)^2\mathrm{E}\left(u_{i,-1}^2\right)+\sigma_\varepsilon^2\frac{\left(1-\rho^{2(T-2)}\right)}{1-\rho^2}-2\rho^{T-2}\sigma_\varepsilon^2+2\sigma_\varepsilon^2\right]\\
&-(1-\rho)\left[\mathrm{E}\left(\alpha_i^2\right)\rho^{T-3}\sigma_\varepsilon^2+\mathrm{E}\left(\alpha_i\right)\rho^{2T-5}\,\mathrm{E}\left(\varepsilon_{i,1}^3\right)\right].
\end{aligned}$$

Recall that the following holds for $A_{n,T}$:

$$A_{n,T}\xrightarrow{p}\frac{\rho\sigma_\varepsilon^2}{1-\rho^2}\left(1-\rho^{T-2}\right),$$

which means that

$$\sqrt{n}(\widehat{\rho}-\rho)\xrightarrow{d}N\left(0,V\right),$$

with corresponding asymptotic variance

$$V=\frac{V_H}{\frac{4\rho\sigma_\varepsilon^2}{1-\rho^2}\left(1-\rho^{T-2}\right)}.$$

If we now let $\rho\to1$, then

$$\begin{aligned}
V_H\to&2\sigma_\varepsilon^2\,\mathrm{E}\left(\alpha_i^2\right)+4\,\mathrm{E}\left(\alpha_i\right)\mathrm{E}\left(\varepsilon_{i,1}^3\right)+2\sigma_\varepsilon^2\,\mathrm{E}\left(u_{i,-1}^2\right)+2\,\mathrm{E}\left(\varepsilon_{i,1}^4\right)\\
&+4\sigma_\varepsilon^4+2\sigma_\varepsilon^4\left(T-3\right)-4\sigma_\varepsilon^4+\sigma_\varepsilon^4\left(T-2\right)\\
&=2\sigma_\varepsilon^2\left(\mathrm{E}\left(\alpha_i^2\right)+\mathrm{E}\left(u_{i,-1}^2\right)\right)+\left(3T-8\right)\sigma_\varepsilon^4+4\,\mathrm{E}\left(\alpha_i\right)\mathrm{E}\left(\varepsilon_{i,1}^3\right)+2\,\mathrm{E}\left(\varepsilon_{i,1}^4\right),
\end{aligned}$$

and $A_{n,T}\xrightarrow{p}\sigma_\varepsilon^2\left(T-2\right)$. We obtain the asymptotic distribution for the unit root case as a result. This shows that there is no discontinuity in the limit theory at $\rho=1$.

When we let $T \to \infty$,

$$\begin{aligned}
V_H &\to 2\sigma_\varepsilon^2 \operatorname{E}\left(\alpha_i^2\right) + 2\operatorname{E}\left(\alpha_i\right)\operatorname{E}\left(\varepsilon_{i,1}^3\right) + \operatorname{E}\left(\varepsilon_{i,1}^4\right) + \sigma_\varepsilon^4 + 2\sigma_\varepsilon^4 \frac{\rho^4}{1-\rho^2} - \sigma_\varepsilon^4 \\
&\quad + \left[(1-\rho)^2 \operatorname{E}\left(\alpha_i^2\right) + \sigma_\varepsilon^2\right]\left[\operatorname{E}\left(u_{i,-1}^2\right) + \sigma_\varepsilon^2 \frac{1}{1-\rho^2} + 2\sigma_\varepsilon^2\right] \\
&= 2\sigma_\varepsilon^2 \operatorname{E}\left(\alpha_i^2\right) + 2\operatorname{E}\left(\alpha_i\right)\operatorname{E}\left(\varepsilon_{i,1}^3\right) + \operatorname{E}\left(\varepsilon_{i,1}^4\right) + 2\sigma_\varepsilon^4 \frac{\rho^4}{1-\rho^2} \\
&\quad + \left[(1-\rho)^2 \operatorname{E}\left(\alpha_i^2\right) + \sigma_\varepsilon^2\right]\left[\operatorname{E}\left(u_{i,-1}^2\right) + \sigma_\varepsilon^2 \frac{1}{1-\rho^2} + 2\sigma_\varepsilon^2\right],
\end{aligned}$$

and

$$A_{n,T} \xrightarrow{p} \frac{\rho\sigma_\varepsilon^2}{1-\rho^2}.$$

By the Slutzky Lemma, we have the following sequential limiting distribution

$$\sqrt{n}(\widehat{\rho} - \rho) \xrightarrow{d} N\left(0, V\right),$$

where the corresponding asymptotic variance is

$$V = \frac{V_H}{\frac{\rho\sigma_\varepsilon^2}{1-\rho^2}} = \left(1-\rho^2\right)\frac{V_H}{\rho\sigma_\varepsilon^2}.$$

We find that $\widehat{\rho}$ converges at rate $\sqrt{n}$ whether or not T stays fixed or $T \to \infty$ (under Assumption A4). When $\rho = 1$, we find that there is no discontinuity in the limit theory for fixed-T settings but we have faster than $\sqrt{n}$-convergence for large-T settings.

3 Concluding remarks

We derive asymptotic results for an IV estimator based on the nonlinear Ahn-Schmidt moment conditions in two settings – under large-n, fixed-T and large-n, large-T. Since the Ahn and Schmidt moment conditions produce estimating equations that are nonlinear in parameters, the results shed light on the nature of the roots and which root is the consistent one. We believe that the alternative of looking into a GMM objective function is much less manageable. We have also found that the asymptotic variance of the estimator in the large-T case is substantially lower than the estimator proposed by Han and Phillips (2010) in the unit root case. It is unknown whether our proposed estimator has the smallest asymptotic variance in the class of estimators that converge at rate $\sqrt{nT}$.

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