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Individual Investors and Suboptimal Early Exercises in the Fixed-Income Market

Abstract

This paper is the first to analyze and value early exercises of Individual Investors in fixed-income investment products. Assuming decision and transaction costs we consider that a continuous decision-making on holding or exercising is not optimal anymore and propose a new approach to modeling exercise decisions, which endogenously determines the optimal decision strategy. Calibrating our model to a unique data set of about 880,000 early exercises in non-tradable putable bonds over a time period of 13 years indicates that Individual Investors (i) act very heterogeneously, (ii) behave as if they face significant individual transaction and decision costs, (iii) react sluggishly, and that (iv) exogenous effects such as taxes or investors' desire for liquidity additionally influence early exercise behavior.

Keywords: decision strategy; decision costs; transaction costs; putable bond

JEL classification: G10, G12, G13

1 Introduction

The right to exercise or redeem early before maturity, is a common feature of many investment and credit products for Individual Investors, such as savings bonds, stock and index options, mortgages or other financial innovations. However, while Individual Investors' trading and exercise behavior has been researched extensively with regard to stocks, equity index options and mortgages, surprisingly little is known about Individual Investors' use of options and derivatives on fixed-income markets.[1] This is particularly remarkable as Individual Investors' portfolios comprise interest-earning assets much more often than equity products (for Germany: Statistisches Bundesamt, 2010; for the U.S.: United States Census Bureau, 2010; for the U.K.: Office for National Statistics, 2012).

With this paper we aim to fill this gap in the current literature and present a theoretical and empirical analysis of Individual Investors' early exercise decisions in standard fixed-income products. In doing so we make two major contributions. First, we develop a new rational model to determine Individual Investors' early exercise decisions within the standard framework of risk-neutral derivatives valuation. Second, we present, to the best of our knowledge, the first comprehensive overview of Individual Investors' empirical early exercise decisions in putable bonds (German Federal Saving Notes) over a period of 13 years. Based on this large and unique data set we calibrate and apply our model. Our results provide new insights in Individual Investors' exercise behavior and are—among others—highly relevant for the risk and liquidity management of issuers of fixed-income products.

In general, it is well known in the literature that investors use early exercise rights "suboptimally". They commonly fail to follow the optimal strategy and forgo the theoretically optimal

[1] As mentioned, a notable exception is the broad strand of literature about prepayment rights in mortgages and mortgage-backed securities such as, e.g., Schwartz and Toruos (1989), Schwartz and Torous (1992) and Deng et al. (2000), who propose reduced form models, and, e.g., Dunn and McConnel (1981a,b), Stanton (1995) and Kalotay et al. (2004), who focus on structural models.

exercise (e.g., Overdahl and Martin, 1994; Dawson, 1996; Pool et al., 2008; Barraclough and Whaley, 2012). Instead, a substantial number of investors exercise early but at a time when it is not rational to do so, or they completely neglect the exercise right (e.g., Gay et al., 1989; Diz and Finucane, 1993; Finucane, 1997; Engström et al., 2000; Poteshman and Serbin, 2003; Liao et al., 2013). In addition, the literature provides some empirical evidence that non-professional investors perform even more poorly in exploiting the option component, compared to other market participants (e.g., Poteshman and Serbin, 2003; Barraclough and Whaley, 2012). The main arguments—besides irrationality—for this suboptimal exercise behavior are individual transaction costs (e.g., Stanton, 1995; Finucane, 1997; Koziol, 2006; Pool et al., 2008), the costs of learning early exercise rules (e.g., Barraclough and Whaley, 2012; Liao et al., 2013), and the effort required to continuously monitor the investment (e.g., Gay et al., 1989; Stanton, 1995; Barraclough and Whaley, 2012; Liao et al., 2013). Regarding transaction costs, it is argued that the premium for exercising early, i.e. the difference between the exercise and the continuation value, must not only be positive for an investor but actually exceed the transaction costs incurred. Consequently, an exercise right might be used more seldom or differently than predicted by standard models with no frictions. Learning and monitoring costs are assumed to lead to non-continuous decision-making, which can result in the neglect of ex-post attractive exercise opportunities and thus in suboptimal early exercises.

Intuitively, in view of such costs, it is a more rational strategy for an investor holding a far out of the money early exercise right to let some time pass by before he “invests” in the next decision regarding holding or exercising, since the currently almost worthless option is not expected to be in the money very soon. On the other hand, if the early exercise right is currently close to the money, the same investor will most likely decide to make the next decision on holding or exercising his option within a short time, because it is more probable that a profitable early exercise opportunity will soon occur. Still, such rational decision strategies have so far rarely been considered in the literature on early exercises of derivatives. Abel et al. (2007), Abel et al. (2009) and Alvarez et al. (2012) investigate "rational inattention" and the implications of "observation costs", but focus on optimal portfolio choices and consumption.[2] Most related to our approach, Stanton (1995) proposes a rational prepayment model for mortgage-backed securities, where mortgage holders do not re-evaluate their portfolio at every possible time but only at random, discrete intervals. However, as a key innovation we replace in our model Stanton's random component with an endogenously determined, optimal strategy. This optimal "decision strategy" defines when to make a decision on continuing to hold or on exercising a financial derivative with an incorporated early exercise right. As part of this strategy, we consider also potential transaction costs if the derivative is exercised and additionally that costs may be incurred for each decision ("decision costs") on holding or exercising.

As a second contribution to the literature, we apply and calibrate our model on a large and unique data set. Over a sample period from July 1996 to February 2009 we examine about 880,000 early exercises of German Federal Saving Notes (GFSN), a simple fixed-income product comparable to a putable bond. Focusing on GFSN allows us to analyze the isolated early exercise behavior of Individual Investors nearly free of any distortions, since these products are almost exclusively sold to Individual Investors and cannot be traded on a secondary market. This means that Individual Investors in GFSN only face the problem of deciding whether to hold or exercise rather than holding, exercising or selling their position as in the case of, e.g., stock options. Hence, in contrast to similar studies such as those by Poteshman and Serbin (2003) or Barraclough and Whaley (2012), who focus on early exercises of exchange-traded options, we do not have to distinguish between different market participants like institutional or retail investors,

[2] A newer strand of literature on executive stock options such as Carpenter et al. (2010) concentrates also on optimal exercise strategies that differ from standard models. Still, modeling a rational decision strategy is not the focus of these studies.

and can base our calibration on a more comprehensive overview of Individual Investors' early exercise decisions.

We observe a very good fit for our model, which we attribute to the endogenous determination of the optimal exercise strategy that—in contrast to pure empirical approaches (such as the reduced form models of Schwartz and Toruos, 1989; or Schwartz and Torous, 1992)—allows also to make rational predictions in changing economic environments, as also pointed out by Stanton (1995). Moreover, we stress four findings from the calibration: first, our results suggest that Individual Investors act heterogeneously, which is in line with the observations of, e.g., Stanton (1995) for mortgage holders or Koziol (2006) for warrant holders, and with the general characterization of Individual Investors' behavior by Barber and Odean (2011). Second, a large number of Individual Investors act as if they face significant transaction and decision costs. Third, many investors behave sluggishly, i.e. they do not exercise at the optimal exercise opportunity, but with a delay. Yet another significant group of "passive investors" never make use of their early exercise option, even when it seems strongly advantageous to do so, which is equivalent to very high transaction or decision costs. Fourth, we identify a stable base exercise rate, independent of any market movements, and observe some unusual peaks in the empirical data, which we attribute to both exogenous effects such as need for liquidity or tax-induced early exercises. Based on our findings we conclude, like, e.g., Poteshman and Serbin (2003) or Barraclough and Whaley (2012) in the case of equity options, that Individual Investors' early exercise decisions in the fixed-income market can clearly differ from the decisions of institutional investors, which results in a present value advantage due to lower option costs for issuers. For our sample we estimate this difference between the standard model value and the empirical value at more than 2%, which is in line with the studies of Stanton (1995) or Koziol (2006), among others.

The remainder of the paper is organized as follows. In Section 2, we present our model and investigate optimal decision and exercise strategies as related to transaction and decision costs. Section 3 introduces our data set and presents summary statistics on the empirical early exercises of Individual Investors. In Section 4, we calibrate the model, test the robustness of our results and estimate the financial advantage for issuers derived from the diverging exercise strategies of Individual Investors. Section 5 concludes and discusses policy implications.

2 Model

In the following, we consider a derivative with an American early exercise right, for which payoff at early exercise is reduced by transaction costs and costs arise with every decision to hold or exercise the derivative ("decision costs"). Since the latter implies that decision-making is costly for an investor, a continuous decision-making strategy for holding or exercising is no longer worthwhile. Instead, a rational investor defines after each decision based on all current information at which upcoming point in time it would be best to make the next decision on the derivative. This endogenous determination of the optimal "decision strategy" is a key component of our model.

2.1 Model setup

We consider a continuous-time economy over the time period $[0, T]$. Let $(\Omega, \mathscr{F}, (\mathscr{F}_t)_{0 \leq t \leq T}, Q)$ be a probability space equipped with a filtration $(\mathscr{F}_t)_t$ fulfilling the usual conditions, i.e. it is right-continuous and complete. Q is the equivalent spot martingale measure where the money market account serves as numéraire. The adapted short rate process is denoted by $r(t)$. We discretize time into equidistant time steps $0 = t_0, ..., t_N = T$ with $t_{i+1} - t_i = \Delta t$. Let $\mathbb{T} = \{t_0, ..., t_N\}$.

The investor holds a derivative X whose payoff at maturity T is given by X_T.[3] Additionally, the derivative can be exercised early in $t_1, ..., t_{N-1}$ with a respective payoff X_{t_i} in t_i. Note that we do not have to specify the type of underlying (e.g., stock or bond) and its dynamics. The only condition we require is that the payoff of the derivative X_{t_i} in t_i is $\mathscr{F}_{t_i}$-measurable, i.e. given the information up to t_i the early exercise payoff or final payoff on the respective date is known, and integrable. Transaction and decision costs in t_i are denoted by TC_{t_i} and DC_{t_i}, respectively, and are also $\mathscr{F}_{t_i}$-measurable and integrable.[4]

Decision strategy

Our first step is defining the points in time at which the investor decides to make a decision on the exercise. We call a non-decreasing sequence $\Gamma = (\Gamma_n)_{n=0,...,N}$ of $(\mathscr{F}_{t_i})$-stopping times $\Omega \to \mathbb{T}$ a *decision strategy* and each Γ_n a *decision point*, if the sequence fulfills the following conditions:

$$\Gamma_0 = 0, \tag{1}$$

$$\Gamma_n < \Gamma_{n+1} \text{ if } \Gamma_n \neq t_N, \tag{2}$$

$$\Gamma_{n+1} \text{ is } \mathscr{F}_{\Gamma_n}\text{-measurable.} \tag{3}$$

Condition (1) simply means that we start today. Condition (2) says if a decision is made at decision point Γ_n the next decision point lies in the future, given that Γ_n is not yet the end of the considered period.[5] In condition (3), $\mathscr{F}_{\Gamma_n}$ is the σ-algebra associated with the stopping time

[3] For the sake of brevity and ease of notation, we do not consider payoffs from the derivative here, such as coupon payments from bonds, before exercise or maturity. However, these can easily be integrated.

[4] Transaction and decision costs can be expressed as a fixed sum or as a relative value. Throughout this paper we work with absolute cost levels, however, our methodology can easily be applied with relative cost definitions.

[5] As $\Gamma = (\Gamma_n)$ is a non-decreasing sequence, condition 2 implies that if $\Gamma_n = t_N$ the following decision points Γ_{n+k} also have the value t_N. Note that these Γ_{n+k} do not provide any new information and could be omitted. However, allowing the sequence Γ to have always the same length simplifies notation here.

Γ_n representing the information available at Γ_n.[6] This condition means that the next decision point Γ_{n+1} is known at the decision point Γ_n. For example, today the investor determines the next decision point Γ_1. Next, in Γ_1 based on the available information at that point in time, he determines Γ_2, i.e. when to make the next decision, etc.

Exercise strategy

Given a decision strategy Γ, the derivative can only be exercised at the decision points Γ_n based on the information associated with Γ_n, yielding a payoff of X_{Γ_n}. Exercising the derivative therefore means stopping the process $(X_{\Gamma_n})_n$. Based on the decision strategy Γ, we call a function $\tau^\Gamma : \Omega \to \{1, ..., N\}$ an *exercise strategy* if it is a $(\mathscr{F}_{\Gamma_n})$-stopping time.

Strategy

The investor has to jointly choose both the decision and the exercise strategy. Therefore, we call a pair $\left(\Gamma, \tau^\Gamma\right)$ a *strategy* that consist of a decision strategy Γ and a respective exercise strategy τ^Γ. The point in time τ where the investor exercises the derivative is then given by:

$$\tau = \Gamma_{\tau^\Gamma}. \tag{4}$$

Note that τ is an $(\mathscr{F}_{t_i})$-stopping time.[7]

Optimal strategy and value

Given a strategy $\left(\Gamma, \tau^\Gamma\right)$, the investor's cash flow consists of the exercise or final payoff X_τ, the transaction costs TC_τ and the decision costs DC_{Γ_n} occurring at the decision points until exercise

[6] I.e. $\mathscr{F}_{\Gamma_n} = \{A \in \mathscr{F} : A \cap \{\Gamma_n \leq t_i\} \in \mathscr{F}_{t_i}$ for all $t_i \in \mathbb{T}\}$.

[7] Note that any strategy determines via (4) a unique $(\mathscr{F}_{t_i})$-stopping time (up to null-sets). On the other hand, any $(\mathscr{F}_{t_i})$-stopping time τ can be represented via (4), for example by setting $\Gamma_n = t_n$ and $\tau^\Gamma = \tau$. However, this representation is not necessarily unique as there may be different strategies inducing a specific τ via (4).

or maturity, i.e. in $\Gamma_1, ..., \Gamma_{\tau^\Gamma}$. We define the sum of the discounted cash flows as $X_{\left(\Gamma,\tau^\Gamma\right)}$:

$$X_{\left(\Gamma,\tau^\Gamma\right)} = \exp\left(-\int_0^\tau r(s)ds\right)(X_\tau - TC_\tau) - \sum_{i=1}^{\tau^\Gamma} \exp\left(-\int_0^{\Gamma_i} r(s)ds\right) DC_{\Gamma_i}, \tag{5}$$

where the first summand of (5) is the discounted payoff from the derivative minus the transaction costs occurring from the exercise in $\tau = \Gamma_{\tau^\Gamma}$. The second summand represents the discounted decision costs that result from the τ^Γ decisions made until the early exercise or maturity date in τ. Given a specific strategy $\left(\Gamma, \tau^\Gamma\right)$, standard pricing theory implies that the "value" of the position equals $E^Q\left(X_{\left(\Gamma,\tau^\Gamma\right)}\right)$.

The rational investor aims at choosing a strategy that maximizes his wealth position. This means he chooses a strategy $\left(\Gamma, \tau^\Gamma\right)^{opt}$ such that:

$$E^Q\left(X_{\left(\Gamma,\tau^\Gamma\right)^{opt}}\right) = \sup_{\left(\Gamma,\tau^\Gamma\right)} E^Q\left(X_{\left(\Gamma,\tau^\Gamma\right)}\right) \tag{6}$$

holds. Accordingly, we get $V_0 = E^Q\left(X_{\left(\Gamma,\tau^\Gamma\right)^{opt}}\right)$ as the value of the derivative in $t = 0$.

In Appendix A we prove the existence of an optimal strategy $\left(\Gamma, \tau^\Gamma\right)^{opt}$, show how it can be constructed and show that the value of the derivative can be calculated via backward induction. We continue with a solution for the optimal strategy. First, we provide the results on the construction of the value process V_{t_i} via backward induction:

- In t_N, i.e. at maturity, set:

$$V_{t_N} = X_{t_N} - TC_{t_N} - DC_{t_N}. \tag{7}$$

- Let the value of the derivative be known for each $t_k > t_n$. Then the value in t_n is:

$$V_{t_n} = \max\left(X_{t_n} - TC_{t_n}, CV_{t_n}\right) - DC_{t_n} \tag{8}$$

$$\text{with} \quad CV_{t_n} = \max_{t_k > t_n} E^Q\left(\exp\left(-\int_{t_n}^{t_k} r(s)ds\right) V_{t_k} \middle| \mathscr{F}_{t_n}\right), \tag{9}$$

where CV_{t_n} is the "continuation value" in t_n.

- In $t_0 = 0$ set:

$$V_0 = CV_0. \tag{10}$$

Based on this, the optimal strategy $\left(\Gamma, \tau^\Gamma\right)^{opt}$ can be constructed as follows:

- In $t_0 = 0$ set:

$$\Gamma_0 = 0. \tag{11}$$

- Assume that the decision point Γ_n has been constructed. Then the next decision point is:

$$\Gamma_{n+1} = \begin{cases} \min\left(t_i > \Gamma_n : CV_{\Gamma_n} = E^Q\left(\exp\left(-\int_{\Gamma_n}^{t_i} r(s)ds\right) V_{t_i} \middle| \mathscr{F}_{\Gamma_n}\right)\right) & \text{for } \Gamma_n < t_N \\ t_N & \text{for } \Gamma_n = t_N. \end{cases} \tag{12}$$

- The optimal exercise strategy is given by:

$$\tau^\Gamma = \min\left(\{i : X_{\Gamma_i} - TC_{\Gamma_i} \geq CV_{\Gamma_i}, \Gamma_i < t_N\} \cup \{i : \Gamma_i = t_N\}\right). \tag{13}$$

As already stated, a formal proof can be found in Appendix A. In the following, we briefly describe the procedure and provide economic intuition for the results.

Starting at maturity (7), the value of the derivative equals its payoff minus transaction and decision costs. In our following analyses, we usually set these costs at zero, i.e. $TC_{t_N} = DC_{t_N} \equiv$

0, which we find most plausible, as the payoff at maturity is usually not an investor's decision and as it is usually cost-free. For all other points in time t_n, equation (8) shows that the value equals the maximum of the immediate exercise payoff less transaction costs $X_{t_n} - TC_{t_n}$ and the continuation value CV_{t_n}, i.e. the derivative value given it is not exercised in t_n, minus decision costs. This is a standard procedure in rational decision-making, also applied in standard American option pricing theory: investors exercise a derivative if—and only if—the immediate exercise value minus transaction costs is larger than the continuation value. Note that while the transaction costs in t_n influence the exercise decision in t_n, the decision costs in t_n do not, as they occur anyway since a decision on the exercise is made.

The key difference of our model to standard American option pricing is the structure of the continuation value CV_{t_n} (see 9) that equals the maximum of the conditional expectations of discounted values in $t_k > t_n$: $E^Q\left(\exp\left(-\int_{t_n}^{t_k} r(s)ds\right) V_{t_k}\middle|\mathscr{F}_{t_n}\right)$. This is highly intuitive for the following reason: let us assume the investor is in t_n and he is not exercising the derivative, which implies that he has to make another decision at the next decision point. As he aims at maximizing his wealth, he will choose that t_k as the next decision point that maximizes the value of his position in t_n. Note that the value of the derivative, given t_k is the next decision point, is $E^Q\left(\exp\left(-\int_{t_n}^{t_k} r(s)ds\right) V_{t_k}\middle|\mathscr{F}_{t_n}\right)$, so that the investor will choose that t_k that maximizes this expression. This can also be seen in the structure of the optimal decision strategy (12): the optimal next decision point is the first upcoming point in time when the respective expected discounted value equals the continuation value. Finally, equation (13) expresses that the derivative should be exercised at the first decision point at which the exercise value minus transaction costs is not smaller than the continuation value or at maturity.

It is clear that the optimal decision strategy strongly depends on decision costs. For example, if the decision costs are very high at any point in time before maturity, it would be optimal to forgo the early exercise opportunity and to choose maturity as the next decision point. In this case the value of the derivative would equal the value of its European version. If all decision costs are zero, i.e. decision-making is cost-free, it would be optimal for an investor to make a decision at each point in time, as he might otherwise miss favorable exercise opportunities by "skipping over" certain dates. This is also implied by our model, as it is straightforward to show that in this case the maximum in (9) is always reached in t_{n+1} so that the continuation value becomes $CV_{t_n} = E^Q\left(\exp\left(-\int_{t_n}^{t_{n+1}} r(s)ds\right) V_{t_{n+1}} \middle| \mathscr{F}_{t_n}\right)$ implying $\Gamma_{n+1} = t_{n+1}$ in (12). Here our procedure coincides with standard American option pricing models.

We note that our methodology is not restricted to valuing and investigating fixed-income investments of Individual Investors, as is the focus in the following. Based on the endogenous determination of an optimal decision and exercise strategy, it is also possible to thoroughly assess a wide range of derivatives with early exercise rights with relation to transaction and decision costs, or other financial products where an endogenously modeled decision strategy is required.

2.2 Numerical example and comparative-static analyses

For illustration, we apply our model in the following to analyze a simple fixed-income derivative under transaction and decision costs. We are mainly interested in the influence of both costs on (i) the value of the derivative, on (ii) the optimal decision strategy, where we investigate the average number of decisions and the average duration between two decision points, and on (iii) the timing of early exercises. Furthermore, we examine the influence of changes in the market environment (volatility) on these results.

We consider an exemplary putable bond with a maturity of 7 years and a notional amount of 1 that promises a yearly coupon of 5%, where coupons are accrued and paid at maturity. In addition, an investor has an early exercise right. In case of early exercise $x.y$ years after issuance, he receives the notional amount plus accrued interest, i.e. the exercise value is given by $1.05^x + 0.05y$.[8]

The short rate process $r(t)$ in this numerical exercise is given by the 1-factor-model of Hull and White (1990), i.e. $dr(t) = \kappa(\theta(t) - r(t))dt + \sigma dW(t)$, where κ is the mean reversion speed, σ the short rate volatility, $W(t)$ a standard Brownian motion and $\theta(t)$ defined according to Hull and White (1994) to fit the current term structure (see, e.g., Brigo and Mercurio (2006) for implementation details). In our standard scenario we set $\kappa = 20\%$, $\sigma = 2.5\%$ and assume the term structure to be flat at 5%. The latter implies that the hold-to-maturity value of the bond, i.e. the value of the bond without option, equals par, so the value of the derivative can be no less than 1. We consider possible transaction costs between 0% and 12% and decision costs between 0% and 2.5%, but assume that both costs are constant for an investor over time and that these costs do not occur at maturity.

The valuation according to (7) to (10) is carried out via Monte-Carlo simulation with 10,000 runs for each cost combination and a step size of $\Delta t = 1/12$, i.e. we allow for 84 time steps until maturity using Euler discretization of the short rate process. To compute the conditional risk-neutral expectation values in (9), we basically follow the least square Monte-Carlo approach proposed by Longstaff and Schwartz (2001).[9] However, in contrast to Longstaff and Schwartz (2001) we determine at each step not only the conditional expectation values for the next point in time, but perform regressions for all upcoming steps until maturity, so that we can identify the time point with the respective highest conditional expectation value.[10] To reduce a potential upward bias in our results (see for details Broadie and Glasserman, 1997), we apply the interleaving estimator as introduced by Longstaff and Schwartz (2001) and described in detail

[8]This profile is a simplified version of the type of bonds (Type B GFSN) we analyze empirically in Section 3.

[9]A detailed analysis of this method is provided in Clément et al. (2002).

[10]As basis function for the regression we choose the first four monomials and a constant. Testing different forms of basis function, we find that this choice performs best regarding efficiency and robustness.

in Glasserman (2004).

Figure 1 presents the valuation results dependent on transaction and decision costs.

[Figure 1 about here.]

As expected, we find the highest value for our derivative with approximately 1.06 in case of zero transaction and decision costs. Hence, the early exercise right has a value of $1.06 - 1 = 0.06$, which coincides with the results from standard valuation models. On the other end, with very high transaction and decision costs, the bond value falls to 1, which means the early exercise right is worthless. For transaction and decision costs between these extremes we recognize in general a strong negative impact of both costs on the valuation, whereby the results are more sensitive regarding changes in decision costs. The first economic intuition is that under transaction and decision costs an investor receives only a reduced payoff at an early exercise and must furthermore account for the cumulated decision costs arising at each decision. Consequently, there is a trade-off after every decision between "investing" very soon in another decision on further holding or exercising the bond and alternatively waiting some time but risking missing out on ex-post attractive exercise opportunities. Naturally, an investor chooses here the strategy, as described in (12), that maximizes the value of the early exercise right. To make the effect of transaction and decision costs on this strategy and thus eventually also on the valuation more obvious, we compare the average number of decisions (Figure 2) and the average duration between two decision points ("decision interval", Figure 3) on the optimal decision strategy.[11]

[Figure 2 about here.]

[Figure 3 about here.]

Beginning again with the case of zero transaction and decision costs, we observe very frequent

[11] The respective averages are calculated under the risk-neutral measure Q over all simulated paths.

decisions until the bond is exercised or redeemed[12] and an average decision interval that lies at approximately 1.0 step. Highly plausible, this implies a de facto continuous decision strategy. It is most to the advantage of an investor to make a decision at every possible point in time, which results in the highest value of the exercise right, as seen before. Higher transaction and, in particular, decision costs induce a more selective optimal decision strategy and a lower average number of decisions. Accordingly, the mean decision interval widens with higher costs, i.e. the investor allows on average more time to pass by before he makes a new decision on his investment. This is reasonable, because if, for example, the early exercise right is currently out of the money, the next potentially attractive exercise point—i.e. the point in time where the option is so deep in the money that it can also account for incurring transaction and decision costs—tends to be at a later point in time than without these costs. Overall, the lower average number of decisions and the wider decision interval comprehend and amplify the negative effect of transaction and decision costs on the value, since possibly lucrative exercise opportunities might be skipped over and thus the advantage of the option is reduced. Finally, in the case of very high transaction and decision costs, the average number of decisions approaches 1 and the decision interval converges to the bond's maturity (84 months). The possible premium of early exercising is not expected to exceed the incurring costs at any time and it is optimal for the investor to fully neglect his exercise right and to simply hold the bond until maturity. Hence, the early exercise right has no value for the investor.

Still, an investor's optimal strategy and the valuation depend also on further factors, such as the market environment or product characteristics. In the following, we exemplary examine the sensitivities of our results regarding changes in the volatility.[13] Figure 4 compares the valuation,

[12]Compared to the theoretical maximum of 84 decisions, the average number of decisions is relatively low here, which is mainly due to the assumed significant volatility combined with a long maturity of 7 years, a situation that facilitates many early exercises.

[13]Separate analyses not reported here show that the economic relations regarding changes in the product characteristics are similar to the results of the volatility analysis. Again, the results of our model are very

the average number of decisions and the decision interval for a low (1.0%), a normal (2.5%) and a high volatility (4.0%) scenario, whereby the middle charts correspond to our standard scenario.

[Figure 4 about here.]

Starting with the low volatility scenario, we observe consistently lower values for all combinations of transaction and decision costs compared to our former results. Clearly, this is due to the one-sided opportunity of the early exercise right, which suffers (gains) from a reduced (enhanced) volatility in the market. Correspondingly, the average optimal number of decisions declines, since the probability of an early exercise opportunity that compensates also for the cumulated transaction and decision costs is lower than in a moderate or high volatility environment. We observe a strong sensitivity regarding changes in transaction and decision costs, so already with moderate costs typically no exercise is any longer feasible and the average decision interval approaches maturity. On the other hand, in the high volatility scenario, the value of the early exercise right increases substantially and even in the case of high transaction and decision costs the option is not forfeited. Accordingly, our model suggests here a denser average decision frequency with shorter decision intervals, which is reasonable since in a high volatility environment more opportunities for early exercises come up.

While the sensitivity analyses above focus on value and the optimal decision strategy, it is also interesting to examine the timing of early exercises in relation to transaction and decision costs, i.e. at what point in time the exercise right is used. Accordingly, Figure 5 provides an overview of the modeled cumulative exercise distributions over all simulated 10,000 paths over time for our standard scenario and selected cost parameters.

[Figure 5 about here.]

reasonable.

In the case of zero transaction and decision costs (upper-left sub-figure) we recognize an approximately linear cumulative distribution of exercises that approaches almost 80% at the last time step. This means that for only 20% of all paths, the investor does not use the early exercise right and holds the bond until maturity. Observing steady and constant exercises over time is plausible if there are no costs, since with both flat coupon payments and a flat interest term structure an early exercise tends to be equally attractive throughout the bond's lifetime. Next, under transaction costs (lower-left sub-figures) the cumulative distribution curve is pulled down but remains a similar shape, which implies steady but overall fewer early exercises. This is also reasonable, since transaction costs do linearly reduce the attractiveness of early exercises for each time step and therefore have a consistent influence on exercise rates over time. Taking decision costs into account (right sub-figures), we recognize two effects on the timing of early exercises. First, decision costs lead to a less smooth distribution. As described, with rising decision costs an investor's optimal decision strategy is more selective and focuses on fewer decision points in time. Hence, the formation of clusters and the increasingly stepped exercise distribution corresponds to the mentioned decrease in the average optimal number of decisions. Second, we find a growing period of no exercise activity at the beginning. With increasing transaction and decision costs, the first optimal decision point at which the potential gain from early exercise can also compensate for the incurring costs recedes further towards maturity, since the potential advantage of a one-sided early exercise option naturally grows over time. In summary, transaction and decision costs have a strong impact on the timing of early exercises and the valuation of the respective option. The main reason is that both costs influence an investor's optimal decision strategy and thus also give rise to different exercise decisions.

3 Data

3.1 German Federal Saving Notes

We base our empirical analysis on the early exercise behavior of Individual Investors in German Federal Saving Notes ("Bundesschatzbriefe", henceforth GFSN). GFSN are putable bonds issued by the Federal Republic of Germany for governmental financing.[14] These products are very well suited for our analysis for the following reasons. First, GFSN are offered exclusively to Individual Investors and resident institutions serving the public benefit, charitable or religious purposes.[15] Because the latter account for only a very small share of the overall investor group, GFSN are de facto a pure Individual Investor product. Thus focusing on GFSN allows us to observe the isolated early exercise behavior of Individual Investors. We do not have to distinguish between different market participants like institutional or retail investors, as in similar studies such as Poteshman and Serbin (2003) or Barraclough and Whaley (2012), who analyze early exercises of exchange-traded options. Second, there is no secondary market, i.e. GFSN cannot be traded. Individual Investors must only decide whether to hold or exercise, rather than to hold, exercise or sell their position. This gives us a much more comprehensive picture of Individual Investors' early exercise activities compared to studies based on tradable financial products such as warrants or call options, where selling is always an alternative to exercising (e.g., Koziol, 2006; Pool et al., 2008). Third, GFSN are simple, easily comprehensible and standardized mid-term fixed-income products, whose structure and general product features have been unchanged since 1969. New issuances have been offered several times a year, which provides us with sufficient cross-sectional

[14]GFSN accounted for about 11.5% of Germany's overall borrowing in July 1996 with outstanding GFSN of approximately €46.3 billion according to the Deutsche Bundesbank. However, the relevance of GFSN for governmental financing decreased with strongly increased debt levels over the years, and the GFSN share of the overall borrowing sank to about 1.0% in February 2009 with an outstanding volume of about €9.5 billion. In 2012, the German government decided to stop offering products exclusively for Individual Investors and stopped issuing new GFSN due to disproportionately high costs.

[15]All product details are described in German Finance Agency (2012).

and time-series variation to analyze the general structure of Individual Investors' early exercises. Fourth, issuer's credit risk can be neglected for GFSN due to the high creditworthiness of the Federal Republic of Germany.

Two types of GFSN exist: Type A, a yearly coupon-paying step-up bond with a maturity of 6 years and Type B, an accrued-coupon bond with rising yearly coupons and a maturity of 7 years.[16] GFSN are offered at nominal value plus accrued interests, whereby at new launches all current issuances are closed. Both GFSN types are equipped with an early exercise right—a specific American put option—, which allows the investor to reclaim his investment plus the accrued interests and for Type B compounded interests at any time after an initial blocking period of one year.[17]

The main distribution channel for GFSN is direct sale by the German Finance Agency ("Bundesrepublik Deutschland Finanzagentur GmbH"), a state-owned central service provider for Germany's governmental borrowing and debt management. The German Finance Agency offers cost-free debt accounts for Individual Investors and does not charge fees for purchasing and administration nor for redemption of GFSN. Alternatively, GFSN can also be acquired through banks, which however typically charge custody fees for administration. It is possible to transfer GFSN ordered through an intermediary to a cost-free account at the German Finance Agency at any time.

For our study on Individual Investors' early exercise behavior we are able to utilize a large, non-publicly available data set from the German Finance Agency. This data set contains—on a daily and single account basis—all Individual Investor investments and early exercises of GFSN from July 1996 to February 2009 that were booked in the German Finance Agency's debt register

[16] This means Type B has a zero-bond structure. At early exercising or maturity, an investor receives the nominal value plus all accrued coupons.

[17] For both types an additional restriction applies: investors are allowed to reclaim a maximal nominal value of €5,000 within 30 interest days, which we ignore in the following.

accounts (approximately 64% of the overall outstanding volume).

3.2 Summary statistics

There are 102 GFSN issuance dates in our sample period with an average time between issuances of 45 days. At each issuance both Type A and Type B are offered with an identical coupon structure for the first 6 years. Table 1 provides an overview of GFSN coupon structures in comparison to prevailing spot rates at issuance.

[Table 1 about here.]

Typically the offered coupons lie below the corresponding spot rates for the first years but rise above market rates towards maturity. Yet in accordance with changes in market conditions, the coupon structure of new issuances is regularly adjusted, implying a significant variance in coupon offerings over time for both Type A and B.

Table 2 presents further summary statistics on our data set along two dimensions. The left sub-tables review the GFSN issuances in our sample. The right sub-tables focus on Individual Investor accounts with funds placed in GFSN.

[Table 2 about here.]

Regarding GFSN issuances, we find that Type A (Type B) GFSN are held on average in 39,719 (26,192) accounts. Still, as indicated by the lower median of only 29,175 (19,155) accounts and the comparatively high standard deviations, the distribution is positively skewed. Similar variations can be observed in the investment volume per GFSN. While the mean volume per GFSN amounts to approximately €103.220 million (€31.666 million), the median volume lies at €77.778 million (€25.703 million). In a more detailed analysis not reported here, we note that both the number of accounts as well as the tendered volume per GFSN tends to decrease over time. Nevertheless, we find that all GFSN are of significant size and sufficiently similar for an overall comparison. Even the least considered GFSN in our sample period (November 2008) is held in more than 4,000 (4,600) accounts at the German Finance Agency.

Over the sample period 812,750 different Individual Investor accounts held GFSN in the German Finance Agency's debt register. About 13% of these accounts acquired both product types, so that we have in total 558,122 Individual Investor accounts for Type A and 361,141 accounts for Type B GFSN. On average each account invested in 3.617 (4.005) GFSN issuances, but we recognize again a distribution skewed strongly to the right, with a median of 1.000 (1.000) investment per account. In other words, the majority of Individual Investor accounts selected only one GFSN issuance throughout the whole sample period. Remarkably, the corresponding mean and median investment volume per account is very small, with only €21,087 (€9,721) or €9,746 (€3,067) per account respectively, which we attribute to the restriction of GFSN to non-institutional investors.

Finally, we observe that Individual Investors in GFSN make frequent use of their early exercise rights. The lower part of Table 2 shows that more than one-fourth of all accounts in our sample exercised early at least once, which equates to an absolute number of 148,812 (95,604) accounts. On average there occurred 6,156 (3,744) early exercises per GFSN issuance, accounting for a mean early exercise volume per GFSN of €17.868 million (€6.740 million). On a single account level, this corresponds to a mean exercise volume per account and GFSN of approximately €2,849 (€1,661).

In summary, the data set provides us with a large number of GFSN issuances, which consistently attracted a wide range of Individual Investors. We find a very low number of investments per account, rather small investment volumes and significant early exercise activities. Since in this study we are most interested in the latter, the following section focuses on analyzing the early exercise rates of GFSN.

3.3 Early exercise rates

To further examine Individual Investors' exercise activities in GFSN, we start with calculating the early exercise volume and the remaining exercisable volume per day for each GFSN. Subsequently, we aggregate these empirical data points to an *early exercise rate per month* $EER^{\text{Obs}}_{g,t}$ per GFSN by:

$$EER^{\text{Obs}}_{g,t} = 1 - \left(\prod_{d=1}^{D_t} \left[1 - \frac{\text{Early Exercise Volume}_{g,t,d}}{\text{Exercisable Volume}_{g,t,d}} \right] \right), \tag{14}$$

where g is the product index, t the month index, d the day index and D_t stands for the number of days in the respective month.[18]

As outlined in the lower part of Table 2, the mean monthly exercise rate is 0.570% (0.536%) over all exercisable[19] Type A (Type B) GFSN, whereby the high standard deviation of 1.028% (0.810%) points towards a notable variance among products and months. The upper charts in Figure 6, where we plot the observed early exercise rates for all GFSN against time, also illustrate this. We register strong fluctuations with a maximum early exercise rate of about 16.8% (14.1%) for an individual GFSN and a minimum rate close to 0% for some Type A and B GFSN in selected months. Still, we can identify some patterns. For instance, on an individual bond level we find an increased exercise activity in the first month after the blocking period. Moreover, there are two general peaks in the exercise rates around the years 2000 and 2007, but also a longer period of less exercise activity between 2002 and 2006.

[Figure 6 about here.]

[18] As outlined in Section 3.3.1, Individual Investors can transfer (or deduct) GFSN investments from their bank accounts to a cost-free debt account at the German Finance Agency at any time. Thus we have to consider possible daily changes in the exercisable volume in our calculation of monthly early exercise rates.

[19] 178 GFSN (89 Type A, 89 Type B) are exercisable in our sample, since they have left the initial blocking period.

Searching for rationales behind these patterns, we investigate the relationship between the empirical exercise rates and the economic advantage of an exercise in the respective months. To proxy the latter we calculate a "hold-to-exercise ratio" for each time step and GFSN, which we define as the fraction of the present value of a GFSN without early exercise right to its exercise value, i.e.:[20]

$$\text{Hold-to-exercise ratio}_{g,t} = \frac{\text{Present Value of GFSN without option}_{g,t}}{\text{Exercise Value}_{g,t}}. \tag{15}$$

We use this ratio as a first simple indicator for the potential benefit of an early exercise, since a hold-to-exercise ratio above 100% implies that it is not reasonable for Individual Investors to early exercise their GFSN—except for possible exogenous reasons such as, e.g., the need for liquidity. In contrast, an exercise might be beneficial if the calculated ratio lies below 100%, depending on the option value. The middle charts of Figure 6 depict the development of the hold-to-exercise ratios for all GFSN over time. Like in our analysis of exercise rates, we notice a broad variance throughout the sample period but again some general patterns. We find that the hold-to-exercise ratios stay clearly above 100% for most outstanding GFSN in most months, which indicates a low attractiveness of early exercises, although there are also several months where an early exercise seems to be highly advantageous.

The lower charts in Figure 6 combine both upper analyses and show the relationship between the monthly exercise rates and the corresponding hold-to-exercise ratios. Even though Individual Investors obviously do not act homogeneously, we can observe a clear negative correlation here. The higher the hold-to-exercise ratio, the lower the average monthly early exercise rates tend to be. Nevertheless, even in highly unattractive months with substantially enhanced hold-to-

[20] To keep the analysis simple, we neglect the exercise option in determining the GFSN holding strategy here. However, we find that the economic relations are very similar for more complex ratios.

exercise ratios, some investors still opt to reclaim their investments. As outlined later in more detail, we attribute these unexpected early exercises to exogenous reasons such as investors' liquidity constraints or tax optimizations. On the other hand, early exercise rates tend to grow exponentially with decreasing hold-to-exercise ratios. In particular, we find a strong increase as soon as the hold-to-exercise ratio drops below 100%.

At a first glance the described empirical patterns might be surprising, since according to standard option theory, one would expect a significant, uniform jump in the exercise rates as soon as the economic advantage of an early exercise is higher than the present value of holding the GFSN. However, the empirical findings coincide very well with the reasoning behind our model in Section 2.2.1 regarding transaction and decision costs. With transaction costs, early exercises are only attractive for Individual Investors if the hold-to-exercise ratio falls significantly below 100%, since the economic advantage of an early exercise must compensate also for these additionally incurring costs. Moreover, as pointed out in the comparative-static analysis, transaction and decision costs induce a non-continuous decision strategy and lead to clustered and temporarily offset early exercise points, which might be linked to hold-to-exercise ratios that (clearly) undercut 100%.

To further investigate Individual Investors' early exercises and to avoid misleading inferences based on idiosyncratic patterns of selected GFSN issuances, we move from the single bond perspective to a more consolidated overview. Figure 7 exhibits the *average monthly exercise rate* over all exercisable GFSN of both types over time. Additionally, the corresponding German 1- and 10-year spot rates are depicted, indicating the level and shape of the interest term structure at the respective time in Germany.

[Figure 7 about here.]

We stress five observations here. First, we find that the average monthly rates are in large part very similar for Type A and Type B GFSN. The highest rates for both types are observed around July 2000, where on average about 2.1% (1.9%) of the outstanding volume is returned for Type A (Type B) GFSN. Least activity occurs in both cases at the end of 2005 with average exercise rates as low as 0.2% (0.1%).

Second, similar to the analysis on a single bond base, we identify two periods of elevated exercise rates during our sample period which both coincide with rising short- and middle-term interest rates. In general, Individual Investors' exercise rates appear highly sensitive to spot rate movements and valuation changes. We note a distinct trend among Individual Investors towards more (less) early exercise activity in rising (falling) interest environments. As described, this is reasonable, because with increased interest rates the respective hold-to-exercise ratio falls and it might be advantageous for Individual Investors to deduct their funds or just roll their investments into a newer GFSN issuance with higher coupons.

Third, the aggregated view underlines once again that the investor base acts heterogeneously. We find comparably small mean empirical exercise rates, which also differ among months and products. In the case of homogeneous investors with an identical basis for decision-making, we would instead expect exercise rates of either 100% or close to 0% on a single product level.

Fourth, regardless of any interest rate movements, we identify a base exercise rate of about 0.160% over all GFSN per month. Even in times of broadly appreciating GFSN and high hold-to-exercise ratios there are always some Individual Investors, who still decide to recall their investments before maturity. As already mentioned, we consider this continuous base exercise rate to be exogenously triggered. We argue that, for example, individual liquidity requirements might be a reason for steady early exercises. Indeed, since GFSN can be terminated comparatively easily and without any extra fees, a GFSN exercise might be one of the first choices for an Individual Investor with liquidity constraints.

Fifth, we observe small but regular peaks in the exercise rates of Type B products at the end of a year, especially in 1999 and 2006. We attribute these effects to a second exogenous factor, namely the tax optimization of Individual Investors at the end of a year, a well known effect described in the literature (e.g., Badrinath and Lewellen, 1991; Sias and Starks, 1997; Ivkovic et al., 2005; who find that Individual Investors show frequent tax-motivated trading before the year is over). In Germany interest payments to Individual Investors are generally taxed when they are distributed, whereby only the share of interest payments is considered that exceeds a saver's tax allowance. At the beginning of both years 2000 and 2007 this tax threshold was significantly reduced in Germany, which put in particular Type B GFSN at a disadvantage, since due to the zero-bond structure the tax allowance for such products can be utilized only once, whereas for Type A GFSN it is considered for each year's interest payment. Hence, it could have been advantageous for Individual Investors to exercise Type B GFSN early with hold-to-exercise ratios above 100% shortly before the new tax regulations in 2000 and 2007 became effective, so as to exploit tax advantages. We also ascribe the small but regular peaks around December (year-end effect) for Type B GFSN to taxation rules. The same argument applies: since the taxation is not spread over several years as for Type A GFSN, year-end optimizations of personal tax allowances are much more relevant and can be a good reasons for early exercises. In the following, we account for both exogenous effects—base exercise rate and taxation—by introducing control variables.

4 Empirical analysis

4.1 Calibration procedure

In this section we calibrate our model to the data set and derive conclusions about the empirical early exercise behavior of Individual Investors. Assuming that Individual Investors' exercise strategies of GFSN are driven by individual transaction and decision costs, we aim to determine the empirically best-fitting cost levels for our model. We structure the calibration in four steps. First, we define and estimate an interest rate term structure model. Second, we consider heterogeneity of transaction and decision costs among Individual Investors by forming different investor clusters. Third, we account for increased exercise activities after the initial one-year blocking period and for exogenous effects that are likely to be found in the empirical data—as suggested by the analysis in Section 3.3.3—but which are not incorporated in our model. Eventually, we specify the calibration via an optimization algorithm.

Interest rate model

For the valuation of interest rate derivatives, typically no-arbitrage term structure models, such as one- or two-factor HJM models or LIBOR market models, are applied (see, e.g., Hull, 2006). While these models have the advantage of being perfectly consistent with the current interest term structure, they prove to be less suitable for our study due to possible logical violations in comparing the empirical and modeled optimal exercise strategies. With a continuous recalibration it might happen that the optimal exercise point determined by the model is empirically never feasible, a circumstance that would distort the calibration quality and reduce its reliability. Therefore, we choose to apply a more sophisticated model of the class of essential affine term structure models according to Dai and Singleton (2000), which determines the interest term structure throughout the whole sample period based on a time-invariant function of only a small set of common state variables.

Egorov et al. (2006) and Tang and Xia (2007) show that a three-factor essentially affine model, with one factor affecting the conditional variance matrix, provides empirically the best performance for different countries including Germany, while it also allows for very flexible specifications. Accordingly, we utilize for the calibration a three-factor essentially affine model $EA_1(3)$. We calibrate the model to monthly German term structures over the period of July 1996 to February 2009 via a Kalman filtering algorithm together with quasi-maximum likelihood, under the assumption that the rates of all yearly maturities 1 to 10 years are not perfectly observed. This is basically the same procedure as used by, e.g., Hördahl and Vestin (2005). The estimated model parameters are presented in Table 3.

[Table 3 about here.]

Heterogeneity of Individual Investors

As already discussed, it is likely that transaction and decision costs vary among Individual Investors. To incorporate this heterogeneity in the calibration while also preserving a low number of estimation parameters and numerical efficiency, we make three assumptions. First, each investor has constant transaction and decision costs, i.e. a specific Individual Investor features the same cost profile over the whole sample period and for each acquired GFSN. Second, we do not estimate individual costs for each single account but group Individual Investors in discrete clusters, which is a common approach in the literature (e.g., Stanton, 1995; Koziol, 2006) to capture heterogeneity. We allow for 9 different clusters of transaction costs from 0% to 7.5% and for 9 different clusters of decision costs from 0% to 0.75%,[21] which gives us overall 81 clusters representing a wide choice of different combinations of transaction and decision costs. In addition, we consider that there might be a group of "passive investors", who under no circumstances make use of their exercise rights, and form an extra cluster with "infinite" or very high transaction and decision costs. Third, the pool of Individual Investors is identical for each GFSN of the same type with regard to the distribution of transaction and decision costs,

[21] Obviously, there is a broad variety of possible cost ranges we can assign to the clusters. Intensively testing different approaches, we find that the following cost parameters are most suitable for our calibration: decision costs of [0%, 0.010%, 0.025%, 0.050%, 0.0750%, 0.100%, 0.250%, 0.500%, 0.750%] and transaction costs of [0%, 0.100%, 0.250%, 0.500%, 0.750%, 1.000%, 2.500%, 5.000%, 7.500%].

which seems plausible in view of the broad range of investors in each GFSN and the unchanged product structure throughout the sample period.

The key challenge of our approach is the determination of appropriate cluster weights. Avoiding estimating each proportion separately, we assume here—extending the approach of Stanton (1995)—that all cluster weights can be described by a combination of two discretized beta distributions. We choose the beta distribution because it can adapt to a multitude of different shapes based on the estimation of only two parameters α and β.[22] Utilizing the cumulative distribution function of the beta distribution, which is given by:

$$F(x|\alpha,\beta) = \frac{1}{B(\alpha,\beta)} \int_0^x t^{\alpha-1}(1-t)^{\beta-1}dt, \tag{16}$$

where $B(\alpha, \beta)$ stands for the standard beta distribution with parameters α and β, we estimate all individual cluster weights p at issuance through:

$$\begin{aligned} p_{g,0,i,j} = (1-\omega) \times & \left[F\left(\frac{i}{9}|\alpha_{TC},\beta_{TC}\right) - F\left(\frac{i-1}{9}|\alpha_{TC},\beta_{TC}\right)\right] \\ & \times \left[F\left(\frac{j}{9}|\alpha_{DC},\beta_{DC}\right) - F\left(\frac{j-1}{9}|\alpha_{DC},\beta_{DC}\right)\right], \end{aligned} \tag{17}$$

where ω is the proportion of passive investors, g is the GFSN index, i and j are the cluster indices for the respective transaction and decision costs, and the time index 0 denotes that we start at issuance.

We know from our model that there is always only one individual optimal strategy $\left(\Gamma, \tau^{\Gamma}\right)$ for each combination of transaction and decision costs. This means we get diverging optimal exercise strategies among our clusters. For instance, we can presume that clusters with comparatively

[22]To reduce complexity we limit in the following the interval of possible values for α and β to [0,8]. Testing several possible interval ranges we find that this restriction does not have a significant influence on the results.

low transaction and decision costs use their exercise rights more often and earlier than others. Conversely, clusters associated with high costs might take later or no advantage of attractive exercise opportunities. Such different exercise strategies result in shifts in the relative cluster proportions over time and induce a phenomena known as burnout (e.g., Stanton, 1995). Burnout basically refers to the dependency of early exercise rates on former exercise activities, which leads in our case to a declining sensitivity over time of all GFSN regarding interest rate movements due to the shrinking proportion of clusters with low transaction and decision costs, which have not exercised yet.

Besides the burnout feature we also take into consideration that Individual Investors can act sluggishly, which means that not all investors in a cluster exercise at the optimal exercise month.[23] More precisely, we allow that at the optimal exercise point a sluggish proportion s of investors delay the exercise (e.g., because they are lazy or hesitant) and instead defers the transaction to an upcoming month. We assume a latest sluggish reaction after 6 months and thus restrict s to an upper bound of 65%.[24] Consequently, the overall *endogenous* exercise rate EER^r per month and GFSN is given by:

$$EER^r_{g,t,i,j} = \begin{cases} (1-s) \times p_{g,t,i,j}, & t = \Gamma_{\tau^{\Gamma}_{g,i,j}} \\ (1-s) \times p_{g,t,i,j}, & EER^r_{g,t-1,i,j} > 0 \\ 0, & \text{otherwise,} \end{cases} \tag{18}$$

where t stands for the month index and s for the sluggish proportion of investors. Equation (18) says that a cluster exercises at (or shortly after) an endogenously defined optimal point in

[23]Such sluggish reactions are described in the literature on momentum effects, among others. For example Hvidkjaer (2011) finds extremely sluggish reactions of Individual Investors to past returns.

[24]After 6 months, the maximum remaining investor share in a cluster equals $0.65^6 \approx 0.075$. We assume that these remaining investors also exercise 6 months after the optimal exercise point.

time, which is determined by the decision strategy Γ and the respective exercise strategy τ^{Γ}. At all other months the endogenous exercise rate equals zero. We determine $\Gamma_{\tau^{\Gamma}}$ for all GFSN and clusters according to (11) to (13), whereby we follow the same calculation approach as outlined in Section 2.2.2 for the comparative-static analysis—except that we now replace the one-factor short rate model with the described essentially affine interest term structure model.[25]

Blocking period

The right to exercise a GFSN early cannot be used in the first year after its purchase. As a consequence, we observe increased exercise activities in the first month after this initial blocking period, which might, e.g., be attributed to investors' deferred liquidity demand over the previous twelve months or could simply be a reaction to earlier market changes. We account for these extraordinary exercises after the blocking period by introducing a dummy variable EER^b and define $EER^b_{g,t,i,j} = p_{g,t,i,j} \times EER^b$.

Exogenous effects

In accordance with our empirical observations and the study of Stanton (1995), we control for three different exogenous effects in the calibration. First, we incorporate a base exercise rate, which we set to the empirically observed rate of 0.160% per GFSN per month. Additionally, we consider for Type B GFSN a possible year-end effect in December and taxation effects in December 1999 and December 2006. Thus the *exogenous* early exercise rate EER^e per month and GFSN is given by:

$$EER^e_{g,t,i,j} = p_{g,t,i,j} \times (0.0016 + DYE + DT99 + DT06), \tag{19}$$

[25] Since we have to compute for each GFSN and each month not only the respective exercise value but also all possible continuation values, these calculations are still a challenge even for modern computers. Here, there are up to 300 billion path calculations and up to 30 million regressions needed.

where DYE is the the year-end dummy and $DT99$ and $DT06$ are the respective dummies for changes in the tax legislations. Both endogenous (18) and exogenous exercises (19) as well as the increased demand after the initial blocking period EER^b lead to shrinking cluster weights over time, since less Individual Investors stay invested in the respective GFSN. Hence, as last step in our calibration approach, we model the development of each cluster weight through:

$$p_{g,t+1,i,j} = p_{g,t,i,j} - EER^r_{g,t,i,j} - EER^e_{g,t,i,j} - EER^b_{g,t,i,j}. \tag{20}$$

Overall calibration

Combining equations (16) to (20), the overall early exercise rate EER of a single GFSN in a selected month is given by:

$$EER_{g,t} = \frac{\sum_{i=1}^{9} \sum_{j=1}^{9} EER^r_{g,t,i,j} + EER^e_{g,t,i,j} + EER^b_{g,t,i,j}}{\sum_{i=1}^{9} \sum_{j=1}^{9} p_{g,t,i,j} + \omega}, \tag{21}$$

where g is the GFSN index, t the month index, i and j are the cluster indices and ω stands for the group of passive investors. Aiming at minimizing the difference between the modeled exercise rates and the empirical observations, we apply a sequential programming approach that searches for best fitting distribution parameters (α_{TC}, β_{TC}, α_{DC}, β_{DC}) and model factors (ω, s, EER^b, DYE, $DT99$, $DT06$) for each Type A and B GFSN through a multidimensional non-linear minimization algorithm based on the interior-point method. As a measure of the goodness-of-fit we use the mean squared error (MSE) on a single product base, which we define over all 151 months and all investigated GFSN per type as:

$$MSE = \frac{1}{151} \sum_{t=1}^{151} \frac{1}{N_t} \sum_{g=1}^{N_t} \left(EER_{g,t-d(g)+1} - EER^{\text{Obs}}_{g,t-d(g)+1} \right)^2, \tag{22}$$

where EER^{Obs} is the empirical exercise rate for a selected GFSN and month as determined in (14), EER the model-implied exercise rate according to (19), N stands for the number of outstanding and exercisable GFSN at each month and $d(g)$ is a function determining the issuance month of the respective GFSN.[26] We re-run the calibration algorithm until the measured change in the MSE falls below 0.000001%. Finally, to reduce potential biases in the results due to local minima and to test the robustness of our results, we repeat the entire calibration approach 250 times with random start parameters.

4.2 Calibration results

We note that our calibration is not a full general equilibrium specification, since transaction and decision costs—occurring at the decision points and at exercise in our model—may in reality also occur when the hedge portfolio is adjusted over time, affecting the arbitrage reasoning of standard option theory. However, for example (implicitly) assuming equivalence of real and risk-neutral probability measures is a standard approach in calibrating similar models, such as mortgage prepayment models (see, e.g., Stanton, 1995; Stanton and Wallace, 1998). Table 4 provides the calibration results.

[Table 4 about here.]

Focusing first on the full-sample results (left column), we receive a very good fit of the modeled early exercise rates to the empirical data with a mean squared error (MSE) on a single GFSN

[26] We test several measures of the goodness-of-fit (e.g., MSE of the average exercise rate per month per type), but find that an optimization using the MSE on a single product base provides best results.

and month level of approximately 0.0036% for Type A GFSN (0.0025% for Type B), which equates to a mean absolute error of circa 0.2827% (0.2627%). To verify the model fit, we calculate also the mean R^2 value between modeled and empirical exercise rates over all N GFSN according to $R^2 = \frac{1}{N}\sum_{g=1}^{N} 1 - \frac{\text{Variance of monthly prediction error}_g}{\text{Variance of empirical monthly exercise rate}_g}$ and a consolidated R^2 value between the modeled and empirical average monthly exercise rate according to $R^2 = 1 - \frac{\text{Variance of average monthly prediction error}}{\text{Variance of empirical average monthly exercise rate}}$. Both values are remarkably high with about 50% and 79% for Type A and approximately 47% and 73% for Type B GFSN, whereby the second R^2 value is higher due to consolidation effects. For a further analysis of the model fit, the upper part of Figure 8 depicts the consolidated calibration results compared to the mean empirical rates over time.

[Figure 8 about here.]

We note that our model captures different market phases equally well, as the mean modeled and observed rates differ by less than 0.1% for both types for most months. Only at the beginning of the sample period, between 1997 and 1999, does our model output clearly underestimate the empirical data, which we ascribe at least partly to the launch of a new stock market segment in Germany in 1997. This "New Market" broadly attracted Individual Investors and thus presumably also enhanced the exogenous demand for liquidity.[27]

The next part of Table 4 presents the estimated beta distribution factors. The resulting distributions are diagrammed graphically in the lower part of Figure 8. We get mean decision costs of about 0.062% for Type A GFSN (0.138% for Type B) and remarkably higher mean transaction costs of circa 3.993% (3.791%) over all clusters during our sample period, which implies that empirically Individual Investors do not use their exercise rights as predicted by standard models. Instead, they act as if they face significant transaction and decision costs. It

[27] According to the Deutsche Bundesbank, the share of stock investments of Individual Investors' overall capital increased in Germany from circa 8.0% in 1997 to circa 14.5% in 2000.

is not surprising that decision costs are estimated significantly lower here since these costs can accrue several times and have—as seen—a stronger impact on an investor's decision strategy, whereas transaction costs are due only once, at exercise.

Moreover, we find that the estimated beta distributions for decision costs are clearly bent left towards very low or zero costs, whereas the distributions for transaction costs are more even but somewhat skewed to the right. Decision costs are slightly more important for Type B GFSN, which we attribute to the generally higher nominal value of Type GFSN due to the zero-bond structure with accumulated interest payments and to our definition of absolute rather than relative transaction and decision costs. The broad standard deviations of estimated transaction and decision costs imply strongly diverging optimal strategies among investors and over time.

We estimate the share of passive investors at about 56% (53%) of the overall investor group, which expresses that a considerable proportion of investors in GFSN completely neglect the incorporated early exercise right. In addition, the calibration results indicate that approximately 65% (65%) of the investors per cluster respond sluggishly, which is very close to our upper bound of half a year response time and means that a large share of investors show some kind of delay between the optimal decision and the actual exercise.

Regarding exogenous effects, we find that in particular the adjustments in the tax legislation for the year 2000 had a substantial influence on Individual Investors' early exercise decisions. We estimate that these changes account for additional exercises of circa 1.984% of the outstanding volume of Type B GFSN in December 1999. The year-end effect (0.568%) and the change in taxation in 2006 (1.167%) had, according to our calibration, a smaller but still notable impact.

4.3 Robustness

To verify the results, we re-calibrate our model for different sub-samples. The right part (two columns) of Table 4 shows exemplary results for two calibrations, each based on roughly half of the sample period. We find that the estimated parameters for both the transaction and decision cost distributions and for the share of passive or sluggish investors are relatively robust and stay in similar ranges, which validates our former findings. Moreover, while the calibration performs better and is more stable if we incorporate both exercise peaks around 2000 and 2007 and a longer time horizon, the MSE for our exemplary sub-samples remains at a low level of only circa 0.0036% for Type A GFSN (0.0025% for Type B GFSN) and of about 0.0069% (0.0035%) respectively. Accordingly, we compute again convincing sub-sample R^2 values of 86.005% (77.338%) and 70.105% (58.351%) respectively between the average modeled monthly exercise rates and the average empirical rates.

4.4 Valuation

The comparative-static analysis in Section 2.2.2 revealed that transaction and decision costs result in diverging optimal exercise strategies and timings compared to the standard case. Obviously, issuers gain a financial advantage when Individual Investors exercise at other points in time than would, e.g., institutional investors, who follow standard financial rationality. Similarly, issuers benefit when Individual Investors completely forfeit the exercise right or exercise "randomly".

To quantify the advantage for the issuer, we compare the valuation of the option component of a GFSN at issuance according to standard valuation,[28]—assuming all investors exercise optimally with no transaction and decision costs—(*model value*) with the *empirical value* which

[28] As described, standard market valuation gives the same results as our model with no transaction and decision costs.

incorporates the actual exercise behavior of Individual Investors. We calculate the empirical value in two steps. First, for a given GFSN we simulate 10,000 paths and calculate the optimal exercise point for each combination of transaction and decision costs. Second, we utilize the full-sample calibration results of Section 4.4.2 to weight all exercise values according to our estimated cost distributions, whereby we also account for the base exercise rate, the share of passive investors, sluggish reactions and additional exercises for the first month after the initial blocking period and—in the case of Type B GFSN—at the end of the year. We exclude only the one-time tax effects due to changes in the tax legislation because such events cannot be foreseen in a valuation ex-ante. The overall weighted value is our empirical value.

The first two columns of Table 5 show statistics on the model and the empirical value of the option component for all GFSN in our sample. Additionally, the third column exhibits the respective advantage for the issuer, which we calculate simply as the difference between both values.

[Table 5 about here.]

We find that the value of the early exercise right on the issuance date according to standard market valuation amount on average to 2.2433% of the nominal value for Type A and to 2.9859% for Type B GFSN, whereby the higher option value for Type B GFSN can mainly be attributed to the longer maturity. For both types we note a significant variation over time as indicated by option values of 1.1071% (1.3672%) for the 5 percent quantile and 4.0102% (5.2208%) for the 95 percent quantile. However, the empirical value of the early exercise right at issuance is much lower for most products. On average we compute empirical option values of only approximately 0.2009% for Type A GFSN (1.1411% for Type B), whereby the 95 percent quantile lies at only 0.8194% (2.8956%). These significantly lower empirical values are due to a combination of three factors: first, we have a high proportion of passive investors and the empirical value is a weighted average over all investor clusters. Second, transaction and decision costs can lead to missed optimal exercise opportunities. Third, liquidity-driven exercises (base exercise rate) and exogenously motivated exercises (tax effects) can occur at points in time where it is not optimal to exercise according to standard models and where early exercises can actually have a negative effect on the valuation. In fact, for some GFSN this negative effect can even overcompensate the positive influence of early exercises at attractive points in time, which results in negative option costs for the issuer (see the 5 percent quantile in Table 5). Generally, we observe a significant and consistent financial advantage over time for issuers due to Individual Investors' diverging empirical exercise strategies. We determine an average advantage of circa 2.0424% of the nominal value for Type A and circa 1.8449% for Type B GFSN. Similarly, the median advantage accrues to approximately 1.8559% (1.7604%).

5 Conclusions

In this paper, we analyzed Individual Investors' early exercise behavior in fixed-income derivatives. We showed that given transaction and decision costs, continuous decision-making is no longer worthwhile and developed a new approach to model early exercises that endogenously determines a rational decision and exercise strategy in the face of such costs. Based on a comprehensive empirical data set, we applied our model and found a convincing fit between the modeled early exercise rates and the empirical observations. Our results suggest that a large proportion of Individual Investors act as if they face significant but heterogeneous transaction and decision costs, that there is also a notable share of passive investors and that investors' reaction to exercise opportunities is often sluggish. All these findings imply that the optimal timing of early exercises for Individual Investors can clearly differ from that of, e.g., institutional investors.

Following, we derive three policy implications. First, considering transaction and decision costs, the value of a fixed-income derivative with an early exercise right is lower than suggested by standard valuation models. As a result, issuers gain a significant financial advantage in pricing such derivatives without accounting for investors' individual exercise strategies, which we estimate at around 2% of the nominal value for our sample. Second, we reckon that issuers can influence investors' strategies by designing specific derivative structures. In some separate analyses, we find for example that a very steep coupon structure leads to a higher sensitivity towards decision costs and makes decision-making less attractive during much of the GFSN lifespan until maturity. This implies that investors tend to more frequently forgo possible exercise opportunities. Third, issuers or financial intermediaries dealing with Individual Investors must—besides allowing for rational early exercise decisions—also account for a base exercise rate and for potential, unexpected, early exercise peaks in their risk and liquidity management due to exogenous effects such as changes in tax laws.

Finally, we note that our model need not be restricted to analyzing fixed-income derivatives and Individual Investor's exercise decisions as in this study. Our approach can easily be applied to other derivatives, investor groups or overall research questions, where an endogenously determined decision strategy is relevant. For instance, mortgages might be a possible field of application, as it seems reasonable to assume that homeowners also do not continuously make decisions on refinancing or prepaying, but follow similar rational decision and exercise strategies as described in this paper.

Appendix A — Model proof

The main idea of the proof is to transform the problem into an equivalent optimal stopping problem in a process that is indexed by a partially ordered set. Roughly speaking, we replace the question "When shall I make the next decision?" in the ordered set $\mathbb{T}$ with the question "In which direction shall I go next?" in a partially ordered set $\mathbb{I}$. Re-indexing the (discounted) payoff process accordingly allows us to adopt techniques from the theory of sequential stochastic optimization for multi-dimensionally-indexed processes as, for example, presented by Cairoli and Dalang (1996).

To provide some intuition, let us assume that the derivative can be exercised at the points in time 1, 2 and 3, i.e. $\mathbb{T} = \{0, 1, 2, 3\}$. Then, given a realization ω, there is a one-to-one relationship between a decision strategy and a sub-sequence of the set $\mathbb{I} = \{(0), (0, 1), (0, 2), (0, 3), (0, 1, 2), (0, 1, 3), (0, 2, 3), (0, 1, 2, 3)\}$. Today, in 0, the investor has to decide on the next decision point 1, 2 or 3. This decision is represented by (0,1), (0,2) or (0,3), respectively. If he, for example, decides on 1, i.e. he chooses (0,1), he can make the next decision in 2 or 3, represented by (0,1,2) and (0,1,3). Figure 9 shows the resulting possible transformed strategies. For example, making a decision in 1 and 3 means going from (0) to (0,1) and afterwards to (0,1,3). Thus, making a decision regarding the next decision point is equivalent to choosing a path through the decision tree representing all possible realization of a decision strategy. Assigning to each knot $\gamma \in \mathbb{I}$ of the decision tree the (discounted) payoff of the derivative X_γ—including the transaction and decision costs occurring when going from (0) to γ—, finding an optimal strategy is equivalent to finding an optimal path through the tree and then finding the best time to stop the path, i.e. to exercise the derivative.

[Figure 9 about here.]

Definitions and re-indexing

Let $\mathbb{I} = \{(0, \gamma_1, ..., \gamma_m) : 0 < \gamma_m < \gamma_{m+1}, 0 \leq m \leq N\}$ be the set of all increasing sub-sequences of $\mathbb{T}$ whose first element is 0. For $\gamma \in \mathbb{I}$, define $l(\gamma) = \text{card}(\gamma) - 1$, $L(\gamma) = \gamma_{l(\gamma)}$, and $\gamma^{[m]} = (0, \gamma_1, ..., \gamma_m)$ for $m \leq l(\gamma)$ and $\gamma^{[m]} = \gamma$ for $m \geq l(\gamma)$, i.e. $l(\gamma)$ is the length of the sequence γ excluding the first component, $L(\gamma)$ is the last component of γ, and $\gamma^{[m]}$ is the element of $\mathbb{I}$ that is formed by the first $m + 1$ components of γ or equals γ. We define a partial order "$\prec$" on $\mathbb{I}$ via:

$$\gamma \prec \delta \text{ if } \gamma = \delta^{[l(\gamma)]}, \tag{A.1}$$

i.e. γ is smaller than δ if it forms the first components of δ that is called a *successor* of γ. Define $\mathbb{D}_\gamma = \{\delta \in \mathbb{I} : \gamma \prec \delta \text{ and } l(\delta) = l(\gamma) + 1\}$. Elements of $\mathbb{D}_\gamma$ are the smallest elements of $\mathbb{I}$ that are larger than γ with respect to the order $\prec$ and are therefore called *direct successors* of γ. Note that the set of direct successors of γ is empty if and only if the last component of γ, $L(\gamma)$, equals t_N.

We set for each $\gamma \in \mathbb{I}$:

$$X_\gamma = \exp\left(-\int_0^{L(\gamma)} r(s)ds\right)(X_{L(\gamma)} - TC_{L(\gamma)}) - \sum_{i=1}^{l(\gamma)} \exp\left(-\int_0^{\gamma_i} r(s)ds\right) DC_{\gamma_i}, \tag{A.2}$$

$$\mathscr{G}_\gamma = \mathscr{F}_{L(\gamma)}. \tag{A.3}$$

X_γ represents the discounted payoff if the deterministic decision strategy γ is applied and stopped in $L(\gamma)$, and the process $(X_\gamma)_\gamma$, indexed by the partially ordered set $\mathbb{I}$, is adapted to the filtration $(\mathscr{G}_\gamma)_\gamma$.

Transformed strategies and equivalent optimization problem

We call a sequence $\Gamma' = (\Gamma'_n)_{n=0,\ldots,N}$, of $(\mathscr{G}_\gamma)$-stopping points[29] $\Omega \to \mathbb{I}$ a *transformed strategy*, if the sequence fulfills the following conditions:

$$\Gamma'_0 = (0), \tag{A.4}$$

$$\Gamma'_{n+1} \in \mathbb{D}_{\Gamma'_n} \text{ if } \Gamma'_{n+1} \neq \Gamma'_n, \tag{A.5}$$

$$\Gamma'_{n+k} = \Gamma'_n \text{ for all } k \geq 1 \text{ if } \Gamma'_{n+1} = \Gamma'_n, \tag{A.6}$$

$$\Gamma'_{n+1} \text{ is } \mathscr{G}_{\Gamma'_n}\text{-measurable}. \tag{A.7}$$

Define $l(\Gamma') = \inf\{n : \Gamma'_{n+1} = \Gamma'_n\}$ that is a $(\mathscr{G}_{\Gamma'_n})$-stopping time. Intuitively, a transformed strategy is a rule for choosing a path through $\mathbb{I}$ that is stopped in $\Gamma'_{l(\Gamma')}$. A denotes the set of all transformed strategies and A^γ denotes for $\gamma \in \mathbb{I}$ the set of transformed strategies that stop

[29] A stopping point is the vector-valued analogon of a stopping time, i.e. $\{\Gamma'_n \prec \gamma\} \in \mathscr{G}_\gamma$ for all γ.

in γ or one of its successors, i.e. $A^\gamma = \{\Gamma' \in A : \gamma \prec \Gamma'_{l(\Gamma')} \text{ a.s.}\}$. This implies that the first components of Γ' are deterministic and coincide with $(\gamma^{[0]}, \gamma^{[1]}, ..., \gamma^{[l(\gamma)]} = \gamma)$ which means that the path through $\mathbb{I}$ described by Γ' goes through γ.

There is a one-to-one correspondence between strategies defined in Section 2.2.1 and transformed strategies defined above. Given a strategy $(\Gamma = (\Gamma_n)_n, \tau^\Gamma)$, set

$$\Gamma'_n = \begin{cases} (\Gamma_0, ..., \Gamma_n) & \text{on } \{\tau^\Gamma \geq n\} \\ (\Gamma_0, ..., \Gamma_{\tau^\Gamma - 1}, \Gamma_{\tau^\Gamma}, ..., \Gamma_{\tau^\Gamma}) & \text{on } \{\tau^\Gamma < n\}. \end{cases} \tag{A.8}$$

It is straightforward to show that $\Gamma' = (\Gamma'_n)_n$ is a transformed strategy. On the other hand, given a transformed strategy Γ' set,

$$\Gamma_n = \begin{cases} L(\Gamma'_n) & \text{on } \{l(\Gamma') \geq n\} \\ t_N & \text{on } \{l(\Gamma') < n\}, \end{cases} \tag{A.9}$$

$$\tau^\Gamma = l(\Gamma'). \tag{A.10}$$

Then, $(\Gamma = (\Gamma_n)_n, \tau^\Gamma)$ is a strategy.

We define for a transformed strategy Γ':

$$\begin{aligned} X_{\Gamma'} = \exp\left(-\int_0^{L\left(\Gamma'_{l(\Gamma')}\right)} r(s)ds\right) &\left(X_{L\left(\Gamma'_{l(\Gamma')}\right)} - TC_{L\left(\Gamma'_{l(\Gamma')}\right)}\right) \\ &- \sum_{i=1}^{l(\Gamma')} \exp\left(-\int_0^{L(\Gamma'_i)} r(s)ds\right) DC_{\gamma_i}. \end{aligned} \tag{A.11}$$

Note that each $\gamma \in \mathbb{I}$ defines a unique deterministic transformed strategy by setting $\Gamma' = (0, \gamma^{[1]}, ..., \gamma^{[N]})$ that stops in γ. Thus, (A.2) and (A.11) coincide in this case. Due to the one-to-one correspondence between strategies and transformed strategies the optimization problem (6) from Section 2.2.1 is equivalent to finding an optimal transformed strategy Γ'^{opt}:

$$E^Q\left(X_{\Gamma'^{opt}}\right) = \sup_{\Gamma'} E^Q\left(X_{\Gamma'}\right). \tag{A.12}$$

Solving the problem

We solve this problem via backward induction:[30]

- For all γ with $\mathbb{D}_\gamma = \emptyset$ set:[31]

$$Z_\gamma = X_\gamma. \tag{A.13}$$

- If $l(\gamma) < N$, $\mathbb{D}_\gamma \neq \emptyset$ and Z_δ has been defined for all $\delta \in \mathbb{D}_\gamma$:

$$Z_\gamma = \sup\left(X_\gamma, \sup_{\delta\in\mathbb{D}_\gamma} E^Q(Z_\delta|\mathscr{G}_\gamma)\right). \tag{A.14}$$

$(Z_\gamma)_\gamma$ is adapted to $(\mathscr{G}_\gamma)_\gamma$ and a supermartingale with respect to $\prec$.[32] For each γ we equip $\mathbb{D}_\gamma$ with an order according to the size of the last element $L(\delta)$, $\delta \in \mathbb{D}_\gamma$.

Define:

$$D(\gamma) = \inf\left\{\delta \in \mathbb{D}_\gamma : E^Q(Z_\delta|\mathscr{G}_\gamma) = \sup_{\delta'\in\mathbb{D}_\gamma} E^Q(Z_{\delta'}|\mathscr{G}_\gamma)\right\} \tag{A.15}$$

and let $(\Gamma(\gamma))_{\gamma\in\mathbb{I}}$ be a family of random variables defined backward in the following way:

- For all γ with $\mathbb{D}_\gamma = \emptyset$ set:

$$\Gamma'(\gamma) = \left(\gamma^{[0]}, \gamma^{[1]}, ..., \gamma^{[N]}\right). \tag{A.16}$$

- If $l(\gamma) < N$, $\mathbb{D}_\gamma \neq \emptyset$ and $\Gamma'(\delta)$ has been defined for all $\delta \in \mathbb{D}_\gamma$, set:

$$\Gamma'(\gamma) = \begin{cases} \left(\gamma^{[0]}, \gamma^{[1]}, ..., \gamma^{[N]}\right) & \text{on } \{Z_\gamma = X_\gamma\} \\ \Gamma'(D(\gamma)) & \text{on } \{Z_\gamma > X_\gamma\}. \end{cases} \tag{A.17}$$

A simple exercise shows that $\Gamma'(\gamma)$ is a transformed strategy for each γ with $\Gamma'(\gamma) \in A^\gamma$. In the following we show that $\Gamma'((0))$ is optimal in the sense of (A.12). The following theorem is the key.

Theorem 1 *For all $\gamma \in \mathbb{I}$ and $\Delta' \in A^\gamma$*

$$Z_\gamma = E^Q(X_{\Gamma'(\gamma)}|\mathscr{G}_\gamma) \geq E^Q(X'_\Delta|\mathscr{G}_\gamma) \tag{A.18}$$

[30]The procedure and the following proof of Theorem 1 are analogous to Cairoli and Dalang (1996), pp. 210–212, for constructing so-called optimal "accessible stopping points" when a process is indexed by specific finite subsets of $\mathbb{N}^d$.

[31]Note that $l(\gamma) = N$ implies $\mathbb{D}_\gamma = \emptyset$.

[32]It can be shown that $(Z_\gamma)_\gamma$ is Snell´s envelope, i.e. the smallest supermartingale dominating $(X_\gamma)_\gamma$.

holds.

Proof. (A.18) is trivial for γ with $\mathbb{D}_\gamma = \emptyset$. Let $\gamma \in \mathbb{I}$ with $\mathbb{D}_\gamma \neq \emptyset$ and assume (A.18) has been shown for all $\delta \in \mathbb{D}_\gamma$. We first consider the equality in (A.18). By definition, we have for $F \in \mathscr{G}_\gamma$:

$$\int_F Z_\gamma = \int_{F\cap\{Z_\gamma = X_\gamma\}} X_{\Gamma(\gamma)} + \int_{F\cap\{Z_\gamma > X_\gamma\}} Z_\gamma. \tag{A.19}$$

By hypothesis, $Z_\delta = E^Q(X_{\Gamma'(\delta)}|\mathscr{G}_\delta)$ holds for all $\delta \in \mathbb{D}_\gamma$, and by definition $Z_\gamma = E^Q(Z_\delta|\mathscr{G}_\gamma)$ on $\{Z_\gamma > X_\gamma\} \cap \{D(\gamma) = \delta\} \in \mathscr{G}_\gamma$. This implies $Z_\gamma = E^Q(X_{\Gamma'(\delta)}|\mathscr{G}_\gamma)$ on $\{Z_\gamma > X_\gamma\} \cap \{D(\gamma) = \delta\}$ and therefore $Z_\gamma = E^Q(X_{\Gamma'(D(\gamma))}|\mathscr{G}_\gamma)$ on $\{Z_\gamma > X_\gamma\}$. Consequently, the equality in (A.18) holds. The next step is proving the inequality. Let $\Delta' \in A^\gamma$ with $\Delta' \neq \Gamma'(\gamma)$. For all $\delta \in \mathbb{D}_\gamma$, define

$$\Delta'(\delta) = \begin{cases} \Delta' & \text{on } \{\Delta'_{l(\gamma)+1} = \delta\} \\ (\delta^{[0]}, \delta^{[1]}, ..., \delta^{[N]}) & \text{on } \{\Delta'_{l(\gamma)+1} \neq \delta\}. \end{cases} \tag{A.20}$$

$\Delta'(\delta)$ is a transformed strategy with $\Delta'(\delta) \in A^\delta$. As $\Delta' \in A^\gamma$, Δ' goes through γ which implies $\Delta'_{l(\gamma)} = \gamma$. This implies that we either have $\Delta'_{l(\gamma)+1} = \gamma$, i.e. Δ' stops in γ, or $\Delta'_{l(\gamma)+1} \in \mathbb{D}_\gamma$. Also note that $\{\Delta'_{l(\gamma)+1} = \delta\} \in \mathscr{G}_\gamma$ for all $\delta \in \mathbb{D}_\gamma$. Thus, we have for $F \in \mathscr{G}_\gamma$:

$$\begin{aligned} \int_F Z_\gamma &\geq \int_{F\cap\{\Delta'_{l(\gamma)+1}=\gamma\}} Z_{\Delta'} + \sum_{\delta\in\mathbb{D}_\gamma} \int_{F\cap\{\Delta'_{l(\gamma)+1}=\delta\}} Z_\delta && \text{(A.21)} \\ &\geq \int_{F\cap\{\Delta'_{l(\gamma)+1}=\gamma\}} X_{\Delta'} + \sum_{\delta\in\mathbb{D}_\gamma} \int_{F\cap\{\Delta'_{l(\gamma)+1}=\delta\}} X_{\Delta'(\delta)} && \text{(A.22)} \\ &= \int_{F\cap\{\Delta'_{l(\gamma)+1}=\gamma\}} X_{\Delta'} + \int_{F\cap\{\Delta'_{l(\gamma)+1}\neq\gamma\}} X_{\Delta'} && \text{(A.23)} \\ &= \int_F X_{\Delta'}, && \text{(A.24)} \end{aligned}$$

where the first inequality results from the supermartingale property of Z_γ and the second holds by hypothesis. Consequently, the inequality in (A.18) holds. □

Theorem 1 implies that the transformed strategy $\Gamma'((0))$ defined by (A.17) is an optimal solution of (A.12). Thus, $Z_{(0)} = E^Q(X_{\Gamma'((0))}|\mathscr{G}_{(0)})$ equals V_0, i.e. the value of the derivative, and—due to the one-to-one correspondence—an optimal strategy exists.

The final step is to show (via induction) that the optimal transformed strategy $\Gamma'((0))$ is equivalent via (A.9) and (A.10) to the strategy defined by (11) to (13). The equivalence is trivial for

$n = 0$. Assume it holds for n. Let $H = \{(\Gamma_0, ..., \Gamma_n) = \gamma\} \cap \{D(\gamma) = \delta^*\} \cap \{\tau^\Gamma \geq n\} \in \mathscr{G}_\gamma$. As δ^* is a direct successor of γ, its first n elements form γ. By (A.9) we have $\Gamma_{n+1} = L(\Gamma'_{n+1}) = \delta^*_{n+1}$ on H. It suffices to show that δ^*_{n+1} equals the smallest element of $\mathbb{I}$ larger than γ_n that maximizes $E^Q \left(\exp\left(-\int_{\gamma_n}^{\delta_{n+1}} r(s)ds\right) V_{\delta_{n+1}} \middle| \mathscr{F}_{\Gamma_n}\right)$ on H with $\delta \in \mathbb{D}_\gamma$.

First, define for each $\Delta' \in A^\gamma$:

$$\begin{aligned} X^n_{\Delta'} = \exp\left(-\int_{\gamma_n}^{L\left(\Delta'_{l(\Delta')}\right)} r(s)ds\right) &\left(X_{L\left(\Delta'_{l(\Delta')}\right)} - TC_{L\left(\Delta'_{l(\Delta')}\right)}\right) \\ &- \sum_{i=n}^{l(\Delta')} \exp\left(-\int_{\gamma_n}^{L(\Delta'_i)} r(s)ds\right) DC_{L(\Delta'_i)}. \end{aligned} \tag{A.25}$$

$E(X^n_{\Delta'}|\mathscr{G}_\gamma)$ equals the "value" of the derivative in γ_n if the strategy Δ' is applied. By definition, we have $E^Q(Z_{\delta^*}|\mathscr{G}_\gamma) \geq E^Q(Z_\delta|\mathscr{G}_\gamma)$ for all $\delta \in \mathbb{D}_\gamma$ on H. Given (A.18), $Z_\delta = E^Q(X_{\Gamma'(\delta)}|\mathscr{G}_\delta)$ holds, and, thus,

$$E^Q(X_{\Gamma'(\delta^*)}|\mathscr{G}_\gamma) \geq E^Q(X_{\Gamma'(\delta)}|\mathscr{G}_\gamma) \text{ for all } \delta \in \mathbb{D}_\gamma \text{ on } H. \tag{A.26}$$

As the $\Gamma'(\delta)$ and $\Gamma'(\delta^*)$ coincide in their first n elements, this implies:

$$E^Q(X^n_{\Gamma'(\delta^*)}|\mathscr{G}_\gamma) \geq E^Q(X^n_{\Gamma'(\delta)}|\mathscr{G}_\gamma) \text{ for all } \delta \in \mathbb{D}_\gamma \text{ on } H, \tag{A.27}$$

which implies that δ^*_{n+1} fulfills the above required condition. Analogue considerations hold for τ^Γ.

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Tables

Table 1: GFSN coupon structure and spot rates

		Type A and Type B GFSN						Type B GFSN
		Year 1	Year 2	Year 3	Year 4	Year 5	Year 6	Year 7
Yearly coupon in %	Mean	2.745	3.199	3.627	4.034	4.409	4.821	4.944
	Median	2.750	3.250	3.625	4.000	4.250	4.500	4.750
	Min.	1.000	1.500	2.000	2.500	3.250	3.500	3.500
	Max.	4.500	4.750	5.500	6.250	6.750	7.500	7.500
	Std. dev.	0.810	0.811	0.805	0.819	0.856	0.927	0.924
Spot rate at issuance in %	Mean	3.387	3.540	3.700	3.853	3.994	4.122	4.235
	Median	3.429	3.599	3.779	3.898	3.936	4.021	4.117
	Min.	1.909	2.061	2.207	2.383	2.562	2.730	2.882
	Max.	5.119	5.216	5.267	5.293	5.608	5.940	6.202
	Std. dev.	0.821	0.766	0.727	0.709	0.707	0.713	0.724

The table exhibits statistics on the coupon offerings for Type A and B GFSN for all 204 issuances (102 Type A, 102 Type B) in our sample period from July 1996 to February 2009 and on the corresponding German spot rates. For year 1 to 6 identical coupons are offered for Type A and B at each issuance date, whereas the coupons in year 7 are only applicable for Type B. The spot rates represent the term structure of interest rates on listed Federal securities (method by Svensson) at the respective issuance date according to Deutsche Bundesbank.

Table 2: Summary statistics for GFSN data set

German Federal Saving Notes (GFSN)				Individual Investor accounts			
		Type A	**Type B**			**Type A**	**Type B**
Total issuances		102	102	Total		558,122	361,141
Number of investors per GFSN	Mean	39,719	26,192	Number of different GFSN investments per account	Mean	3.617	4.005
	Median	29,175	19,155		Median	1.000	1.000
	p5	8,711	7,639		p5	1.000	1.000
	p95	100,068	54,037		p95	13.000	16.000
	Std. dev.	32,686	17,385		Std. dev.	6.761	8.559
Volume in €m per GFSN	Mean	103.220	31.666	Investment volume in € per GFSN per account	Mean	21,087	9,721
	Median	77.778	25.703		Median	9,746	3,067
	p5	17.139	8.140		p5	511	255
	p95	254.559	66.864		p95	79,897	37,807
	Std. dev.	89.958	22.664		Std. dev.	41,240	27,702
Number of early exercises		547,864	333,232	Number of accounts with early exercises		148,812 *(27%)*	95,604 *(26%)*
Number of early exercises per GFSN	Mean	6,156	3,744	Number of early exercises per account	Mean	0.982	0.923
	Median	5,508	3,718		Median	0.000	0.000
	p5	514	444		p5	0.000	0.000
	p95	13,286	8,275		p95	5.000	4.000
	Std. dev.	4,173	2,355		Std. dev.	3.116	3.331
Early exercise volume in €m per GFSN	Mean	17.868	6.740	Early exercise volume in € per GFSN per account	Mean	2,849	1,661
	Median	13.989	5.053		Median	0	0
	p5	0.780	0.332		p5	0	0
	p95	46.914	17.164		p95	15,850	9,203
	Std. dev.	15.873	6.401		Std. dev.	8,669	5,937
Monthly early exercise rates in % per GFSN	Mean	0.570	0.536				
	Median	0.240	0.283				
	p5	0.100	0.096				
	p95	2.397	1.892				
	Std. dev.	1.028	0.810				

The tables shows summary statistics on the GFSN data set for our sample period from July 1996 to February 2009. Overall 204 GFSN and 812,750 accounts with 881,096 early exercises are considered. The statistics on early exercises are based only on the 178 exercisable GFSN (after the initial one-year blocking period) from July 1996 to February 2008. €m stands for €million.

Table 3: Parameters for essentially affine three-factor model

	δ	β	κ	θ		
Estimated parameters	0.0106	1.0000	0.0365	0.0000	0.0000	15.0218
	0.0002	4.0189	-0.4965	-0.1798	0.3223	40.8385
	0.0000	2.5195	0.4256	-2.2507	1.6917	50.3077
	0.0013					

The table shows the estimated parameters for an essentially affine three-factor interest term structure model $EA_1(3)$ on a weekly basis with one factor affecting the conditional variance matrix for the German market from July 1996 to February 2009. The model is defined as $r_t = \delta_0 + \delta' Y_t$, whereby the dynamics of Y are modeled by $dY_t = K^Q(\theta^Q - Y_{t_i})dt + \Sigma\sqrt{S_t}d\tilde{W}_t$ and S is given by $S_{t,j,j} = \alpha_j + \beta_j' Y(t)$.

Table 4: Calibration results

		Overall 1996-2009		Sub-sample 1996-2002		Sub-sample 2003-2009	
		Type A	**Type B**	**Type A**	**Type B**	**Type A**	**Type B**
Mean squared error in %		0.0036	0.0025	0.0036	0.0025	0.0069	0.0035
Mean absolute error in %		0.2827	0.2627	0.3061	0.3276	0.5059	0.3860
Mean R^2 (on single GFSN level) in %		49.946	46.606	56.989	52.701	52.519	50.091
Consolidated R^2 (mean exercise rates) in %		78.915	72.715	86.005	77.338	70.105	58.351
Beta distribution of	α_{TC}	1.652	1.397	0.666	1.410	1.771	1.746
transaction costs	β_{TC}	0.637	0.607	0.506	0.929	0.754	1.002
Std. dev.		0.247	0.265	0.336	0.268	0.244	0.249
Skewness		-0.838	-0.742	-0.257	-0.354	-0.731	-0.459
Mean transaction costs in %	TC	3.993	3.791	2.911	2.707	3.693	2.909
Median transaction costs in %	TC	2.500	2.500	1.000	1.000	2.500	1.000
Beta distribution of	α_{DC}	0.132	0.350	0.123	0.523	0.275	0.512
decision costs	β_{DC}	0.679	0.687	0.825	1.642	1.421	0.796
Std. dev.		0.275	0.331	0.240	0.241	0.224	0.321
Skewness		1.745	0.647	2.093	1.031	1.629	0.408
Mean decision costs in %	DC	0.062	0.138	0.043	0.054	0.036	0.155
Median decision costs in %	DC	0.000	0.010	0.000	0.010	0.000	0.025
Passive investors in %	ω	56.340	52.741	58.403	58.803	61.786	62.655
Sluggish investors in %	s	65.000	64.955	62.911	64.890	64.509	63.960
Additional exercises first month after blocking period in %	EER^b	2.092	2.426	3.572	3.175	1.947	3.175
Exogenous early exercises	$DT99$		1.984		2.258		
in % (dummies)	$DT06$		1.167				1.271
	DYE		0.568		0.804		0.792

The table shows calibration results for different data samples. The results in the first column are based on a calibration on the whole sample period from July 1996 to February 2009, while for the second and third column the calibration is based only on a sub-sample from July 1996 to December 2002 respectively on a sub-sample from January 2003 to February 2009. For the calibration all exercisable GFSN are valued based on the model of Section 2.2.1 using Monte-Carlo simulation with 10,000 paths with a step size of $\Delta t = 1/12$ (84 steps) and applying least square regression methods. The interest structure is modeled based on an essentially affine interest term structure model on a weekly basis. Accordingly, the calibration parameters are estimated via a multidimensional non-linear minimization algorithm based on the interior-point method with a MSE change tolerance of 0.000001% as stopping criterion. Additionally, the results are checked for local minima by re-running the calibration 250 times with random start parameters. The base exercise rate is fixed to the empirical observed value of 0.160% to ensure robustness of the calibration. The tax-related dummies are only considered for Type B GFSN (DT stands for tax changes, DYE for the year-end-effect).

Table 5: Comparison of standard model and empirical valuation based on calibration

	Value of early exercise right in GFSN in %				Advantage for issuer	
	Standard model (no transaction and decision costs)		Empirical (according to calibration)		Standard model minus empirical valuation	
	Type A	**Type B**	**Type A**	**Type B**	**Type A**	**Type B**
Mean	2.2433	2.9859	0.2009	1.1411	2.0424	1.8449
Median	1.9486	2.7005	0.1165	0.9372	1.8559	1.7604
p5	1.1071	1.3672	-0.1469	-0.1680	1.1741	1.3136
p95	4.0102	5.2208	0.8194	2.8956	3.2183	2.4394
Std. dev.	0.9704	1.2373	0.3134	1.0210	0.6791	0.4108

The table shows statistics on the valuation of the option component of all 178 exercisable GFSN at issuance in the sample period. The left part compares the valuation according to standard market models without transaction and decision costs and the empirical valuation based on the calibration results of Table 4 except the tax dummies. The right part shows statistics on the spread between the standard model and the empirical valuation.

Figures

Figure 1: Valuation of a putable bond dependent on transaction and decision costs

This figure shows the exemplary valuation of a putable bond dependent on transaction and decision costs. The analysis is based on an accrued-coupon bond that offers a yearly coupon of 5% in a zero bond structure over a maturity of 7 years and grants the investor additionally an early exercise right. The interest rate environment is estimated using the 1-factor-model of Hull and White (1990) with $\kappa = 20\%$, $\sigma = 2.5\%$ and θ adjusted to a long-term interest rate of 5%. For the valuation a Monte-Carlo simulation with 10,000 paths with a step size of $\Delta t = 1/12$ (84 steps) and least squares regression methods are applied.

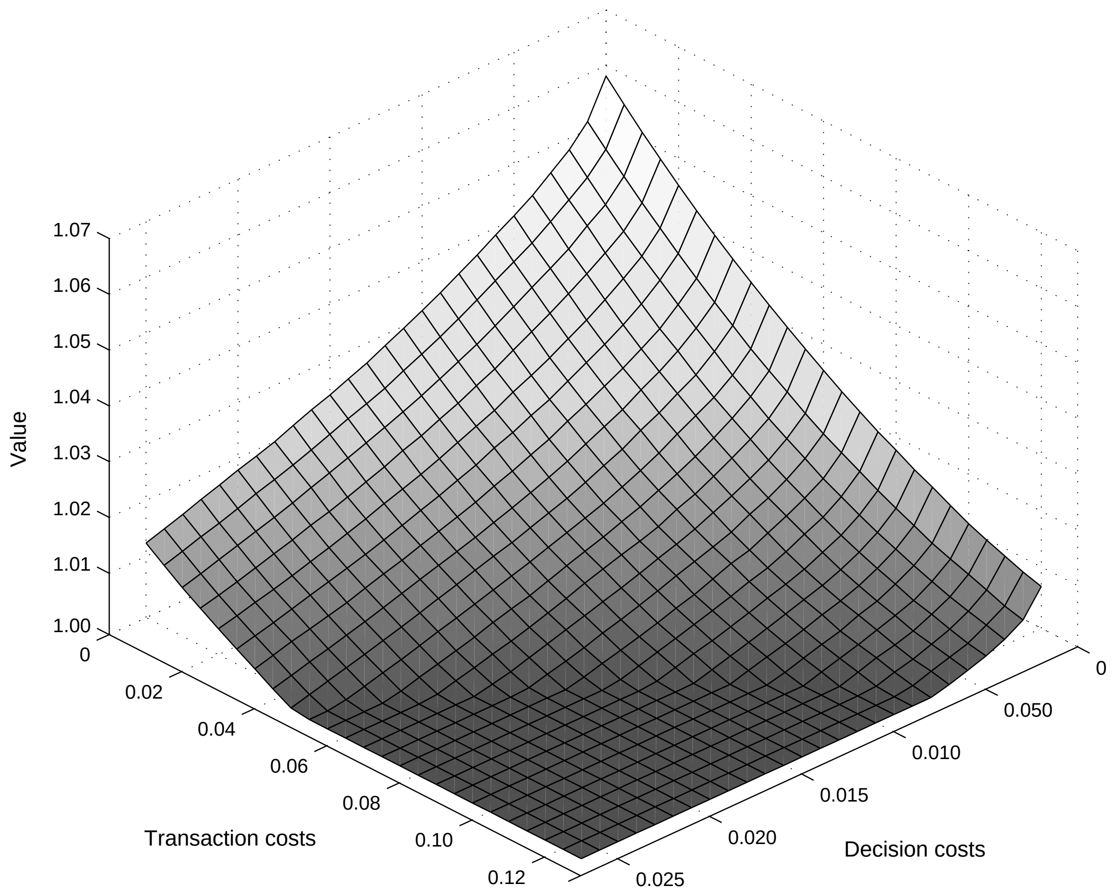


Figure 2: Average number of decisions dependent on transaction and decision costs
This figure shows the average optimal number of decisions according to the optimal decision strategy for an exemplary putable bond dependent on transaction and decision costs. The analysis is based on an accrued-coupon bond that offers a yearly coupon of 5% in a zero bond structure over a maturity of 7 years and grants the investor additionally an early exercise right. The interest rate environment is estimated using the 1-factor-model of Hull and White (1990) with $\kappa = 20\%$, $\sigma = 2.5\%$ and θ adjusted to a long-term interest rate of 5%. For the valuation a Monte-Carlo simulation with 10,000 paths with a step size of $\Delta t = 1/12$ (84 steps) and least squares regression methods are applied, whereby the respective averages are calculated under the risk-neutral measure Q over all simulated paths.

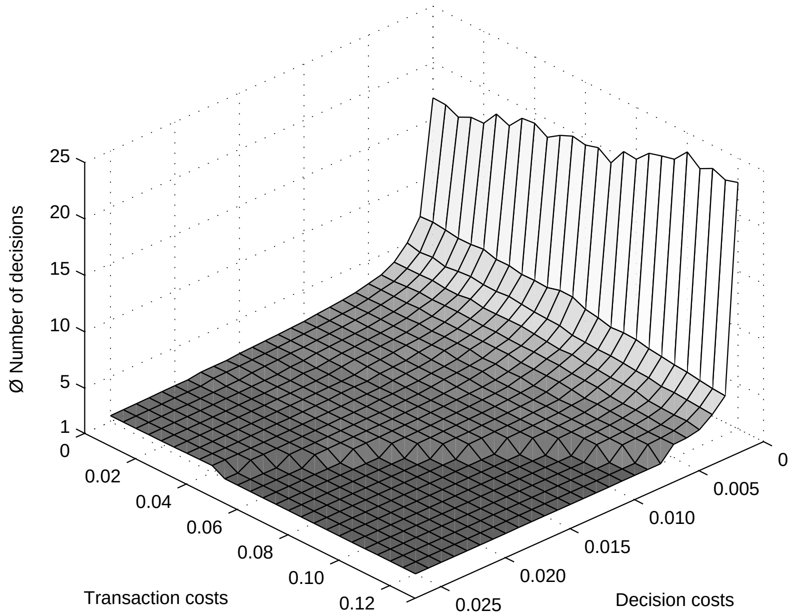


Figure 3: Average decision interval dependent on transaction and decision costs
This figure shows the average duration between two decision points ("decision interval") in steps according to the optimal decision strategy for an exemplary putable bond dependent on transaction and decision costs. The analysis is based on an accrued-coupon bond that offers a yearly coupon of 5% in a zero bond structure over a maturity of 7 years and grants the investor additionally an early exercise right. The interest rate environment is estimated using the 1-factor-model of Hull and White (1990) with $\kappa = 20\%$, $\sigma = 2.5\%$ and θ adjusted to a long-term interest rate of 5%. For the valuation a Monte-Carlo simulation with 10,000 paths with a step size of $\Delta t = 1/12$ (84 steps) and least squares regression methods are applied, whereby the respective averages are calculated under the risk-neutral measure Q over all simulated paths.

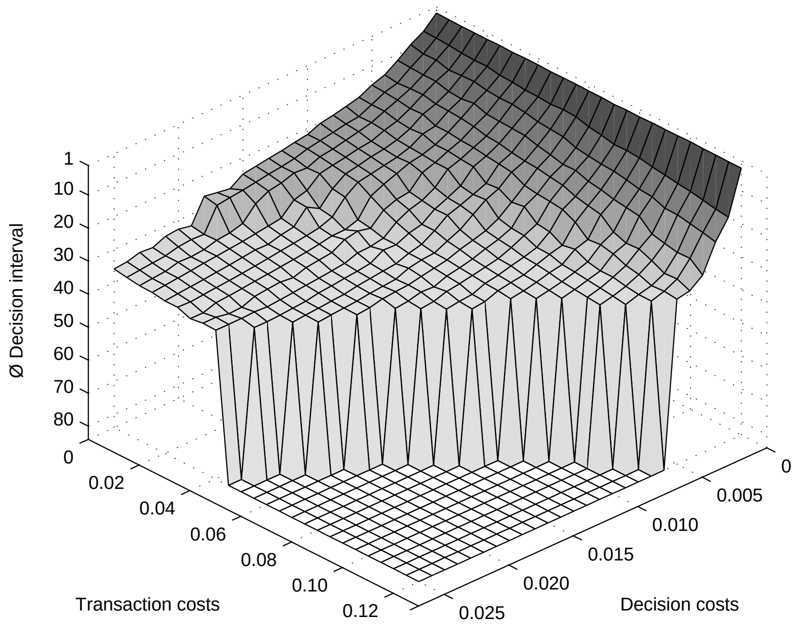


Figure 4: Sensitivities of value, number of decisions and decision interval regarding volatility

This figure presents a sensitivity analysis of the value and the optimal decision strategy, i.e. the average optimal number of decisions and the average decision interval (denoted in steps, minimum 1 step to maximum 84 steps), regarding changes in the interest rate volatility. The analysis is based on an accrued-coupon bond that offers a yearly coupon of 5% in a zero bond structure over a maturity of 7 years and grants the investor additionally an early exercise right. The interest rate environment is estimated using the 1-factor-model of Hull and White (1990) with $\kappa = 20\%$, σ as shown and θ adjusted to a long-term interest rate of 5%. For the valuation a Monte-Carlo simulation with 10,000 paths with a step size of $\Delta t = 1/12$ (84 steps) and least squares regression methods are applied, whereby the respective averages are calculated under the risk-neutral measure Q over all simulated paths.

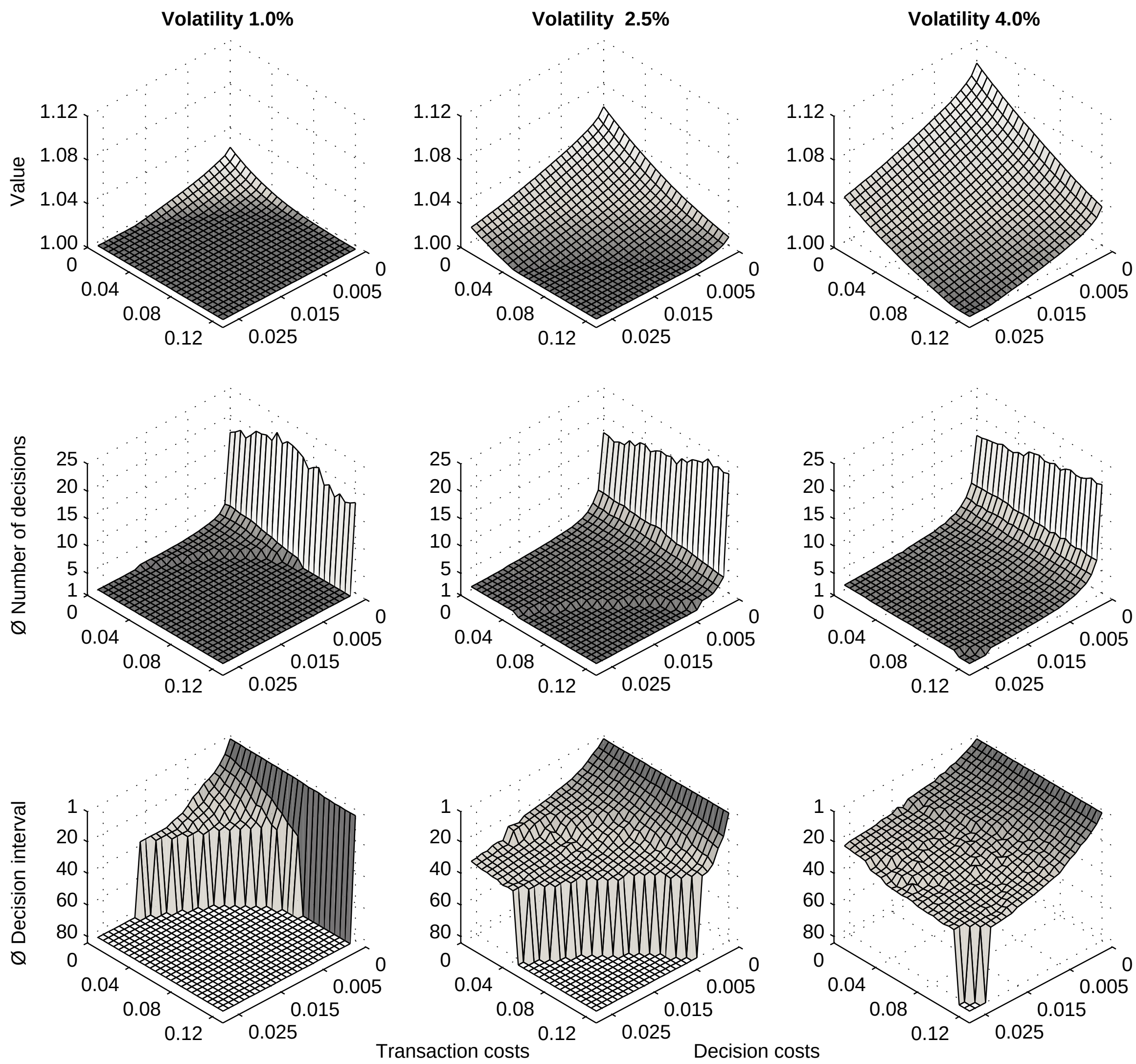


Figure 5: Cumulative exercise distributions over time dependent on selected transaction and decision costs

This figure shows cumulative early exercise distributions for an exemplary putable bond over time dependent on selected transaction and decision costs. The analysis is based on an accrued-coupon bond that offers a yearly coupon of 5% in a zero bond structure over a maturity of 7 years and grants the investor additionally an early exercise right. The interest rate environment is estimated using the 1-factor-model of Hull and White (1990) with $\kappa = 20\%$, $\sigma = 2.5\%$ and θ adjusted to a long-term interest rate of 5%. For the valuation a Monte-Carlo simulation with 10,000 paths with a step size of $\Delta t = 1/12$ (84 steps) and least squares regression methods are applied.

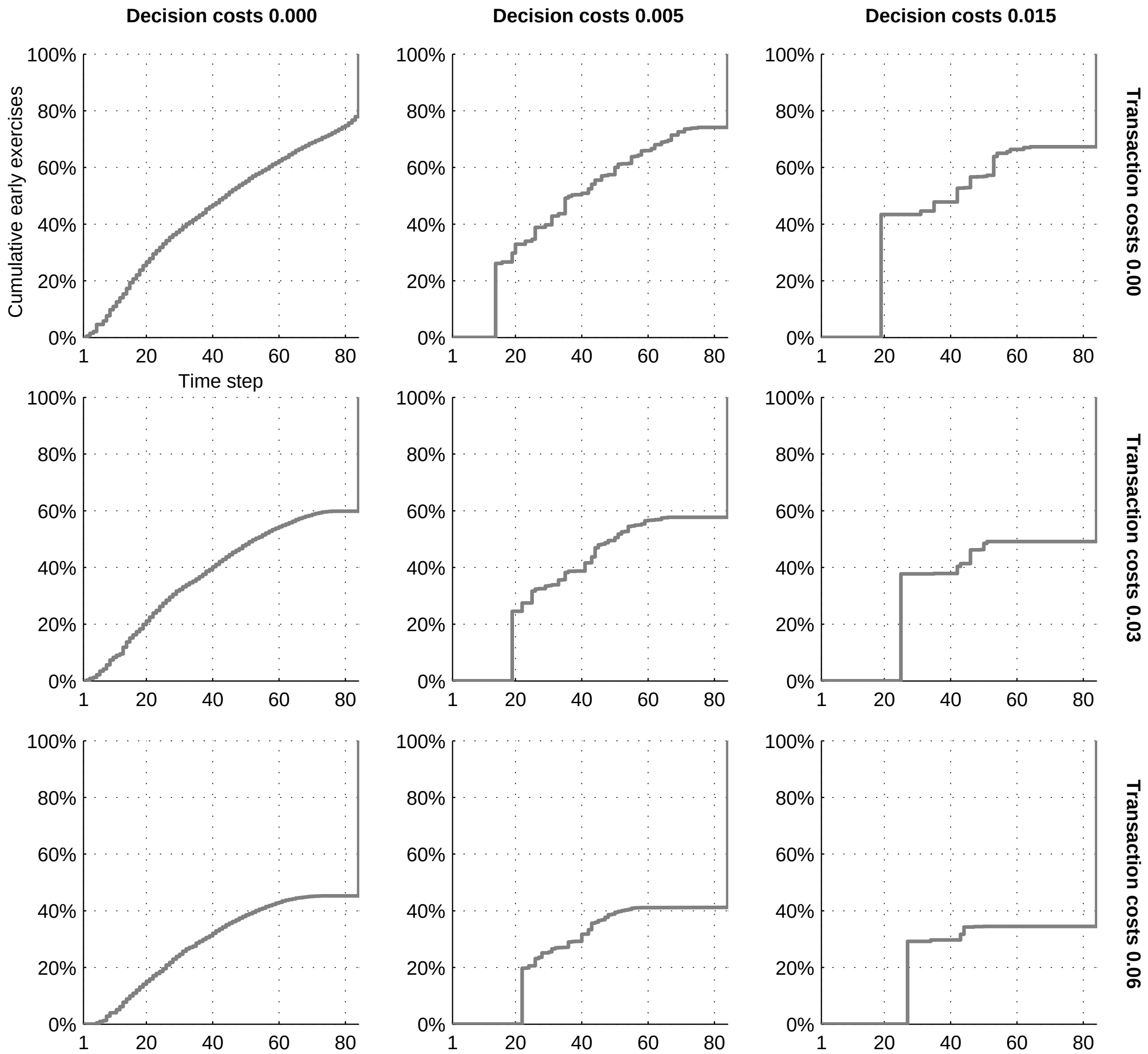


Figure 6: GFSN early exercise rates per month in relation to hold-to-exercise ratio

This figure shows the early exercise rates per month for Type A and B GFSN over time and in relation to the respective hold-to-exercise ratio, which is defined as the fraction of the present value of a GFSN without option to its exercise value. The upper charts present the development of the early exercise rates over time (except for the initial one-year blocking period) for all 178 exercisable GFSN issuances. The middle charts exhibit the development of the hold-to-exercise ratios for all GFSN over time. The lower charts plot the economic relation between the hold-to-exercise ratio and the early exercise rates per month. There are no exercises before 1997 due to the initial blocking period.

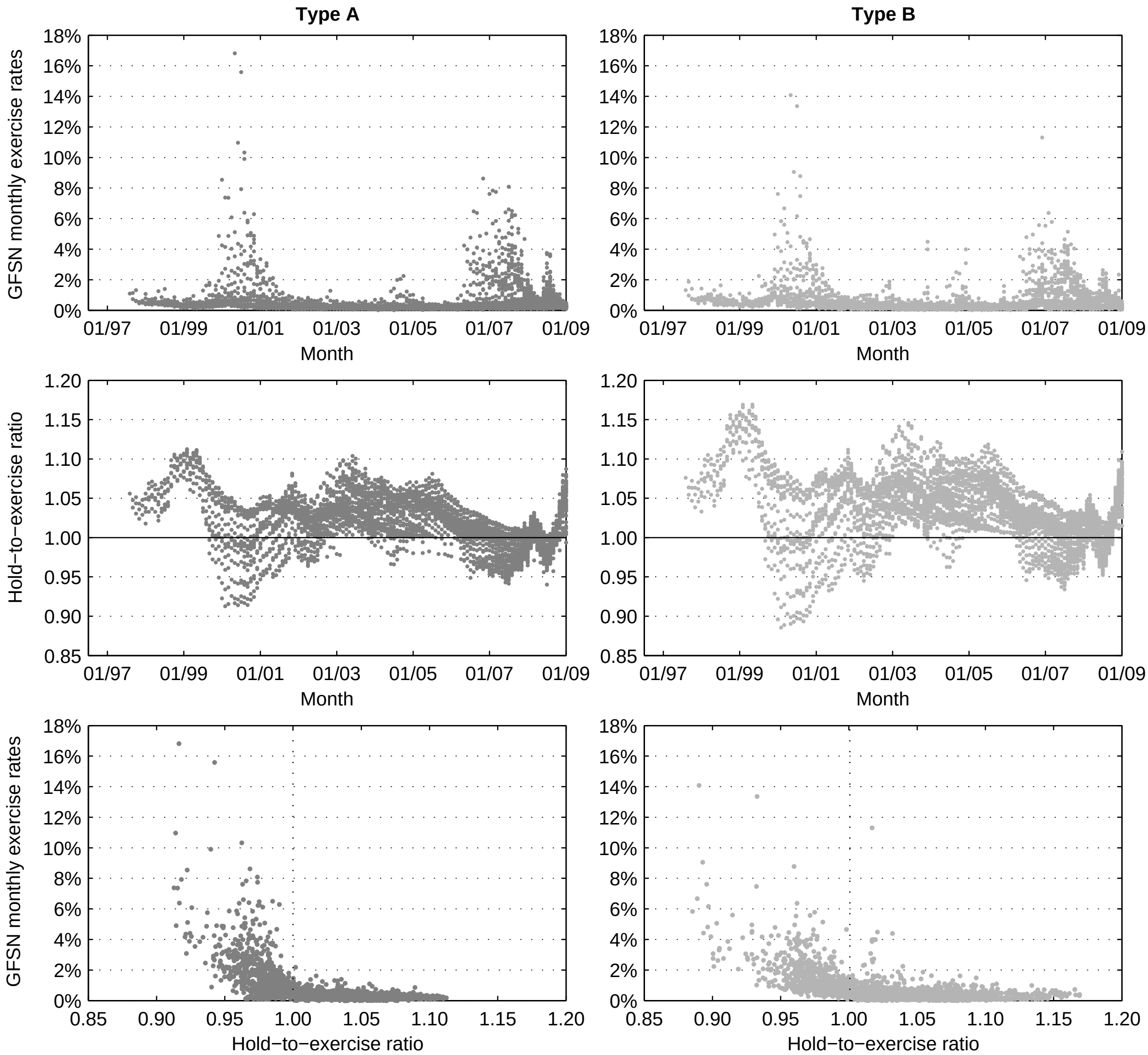


Figure 7: Average monthly GFSN exercise rates
This figure presents the average monthly early exercise rates over all exercisable GFSN of Type A and B from July 1996 to February 2009. There are no exercises before 1997 due to the initial one-year blocking period. Additionally, the development of the respective German 1-year and 10-year spot rates (according to Deutsche Bundesbank) are depicted.

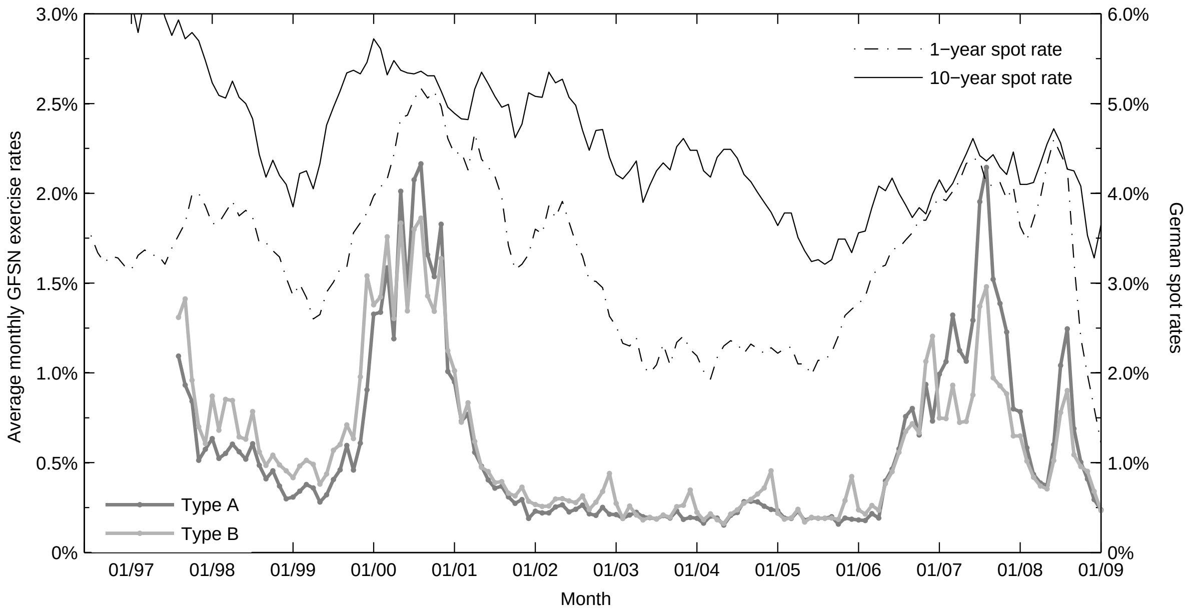


Figure 8: Calibration results and model fit
This figure shows the fit of the overall calibration and the estimated beta distributions for transaction and decision costs for both Type A and B GFSN based on the results in the first column of Table 4. The upper charts present a comparison between the mean modeled early exercise rates per month and the empirical observations. The lower charts exhibit the estimated cost distributions over the defined clusters.

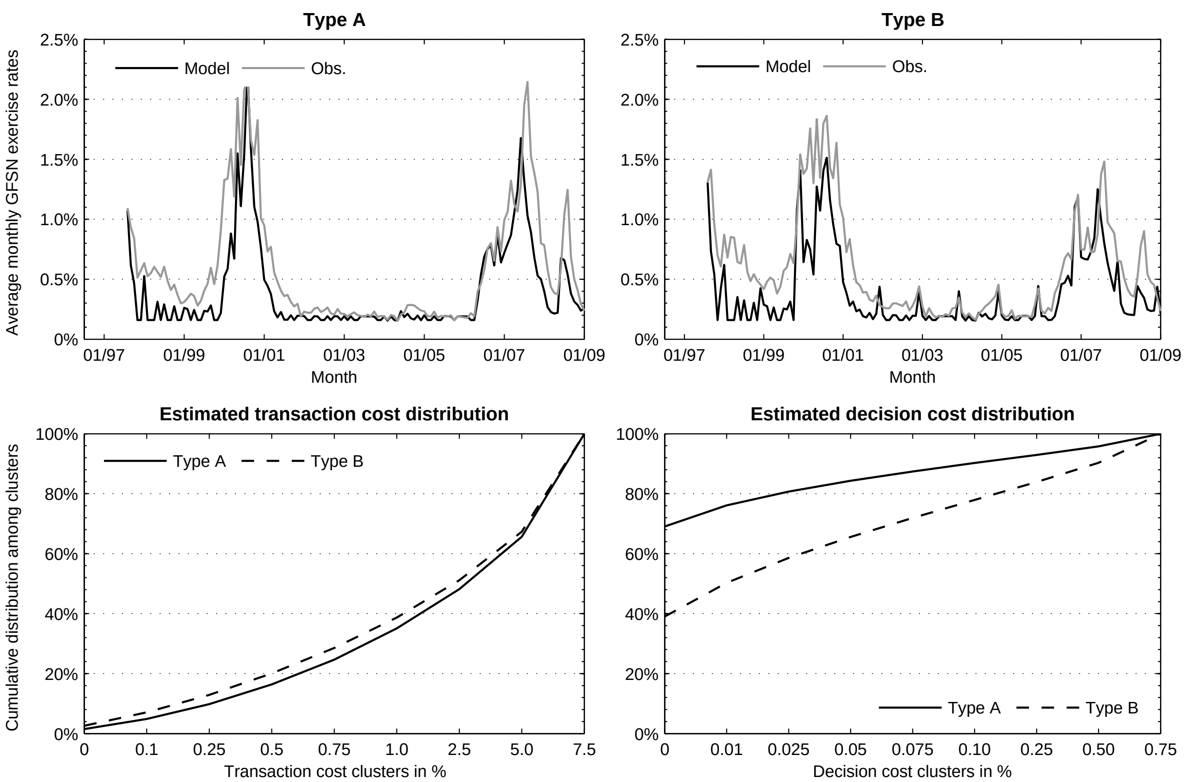
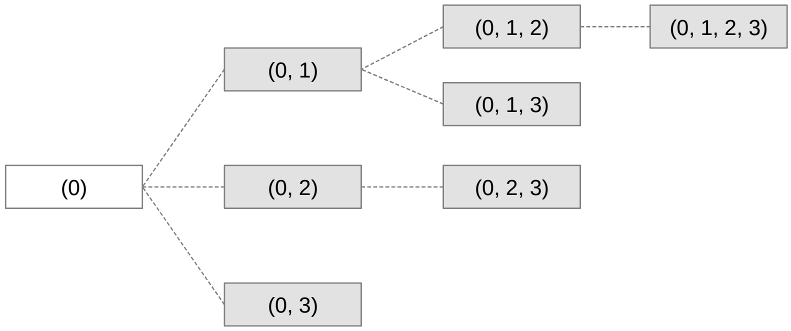


Figure 9: Exemplary decision tree



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