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Portfolio Selection with Time Constraints and a Rational Explanation of Insufficient Diversification and Excessive Trading

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Portfolio Selection with Time Constraints and a Rational Explanation of Insufficient Diversification and Excessive Trading

Armin Dolzer and Bernhard Nietert [*]

Abstract

Private investors have limited time available for learning about stocks as they need to divide their time between stock analysis and work. This paper analyzes the influence of learning constraints in the form of time constraints on portfolio selection and derives both optimal portfolio holdings and time allocation.

Under time constraints, rational private investors make portfolio choices similar to those of investors with bounded rationality, i.e., insufficient diversification and excessive trading. Thus, time constraints offer an alternative, fully rational explanation for these real-world investment phenomena, which have to date been interpreted primarily in the light of behavioral finance.

JEL Classification: G11; G12

Keywords: Excessive Trading; Insufficient Diversification; Learning; Portfolio Selection; Time Constraint

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Portfolio Selection with Time Constraints and a Rational Explanation of Insufficient Diversification and Excessive Trading

1. Preliminaries

1.1. Introduction to the problem

The Internet and financial news on television equip private investors, at no charge, with an abundance of data concerning stocks, including historical stock quotes, companies' fundamental data, and analysts' reports. Moreover, transaction costs for trading are low. Therefore, it comes as no surprise that Goetzmann/Kumar's (2004) empirical study finds that transaction costs, as well as data acquisition costs, do not significantly limit portfolio selection.

However, data cannot be used for decision making; information is required, i.e., messages that are relevant to decision making.[1] Thus, data must be transformed into information so that private investors can use a posteriori instead of a priori distributions of stock prices. This transition from a priori to a posteriori distributions constitutes a learning process and, obviously, learning takes time. Time is a scarce resource and its scarcity is seen as one of the major problems in decision making (see, e.g., Juster/Stafford, 1991; Mankins, 2004). Hence, limited time means learning constraints for decision makers and the question arises as to how the time that is available should be allocated between learning about stocks via stock analysis and other activities such as work.

Starting from this description of the problem, the objectives of our paper are twofold. First, we aim to determine the optimal solution to the portfolio selection and time allocation problems. Second, we want to demonstrate that normative portfolio selection with time constraints

[1] This understanding of information combines Mag's (1977, p. 4) definition of information with the distinction between information and knowledge in Kuhlen (1995, p. 38).

can be applied to explain two real-world investment phenomena, namely, insufficient diversification and excessive trading. To date, these investment phenomena have been interpreted primarily in the light of behavioral finance.

To achieve these two objectives, we consider a learning process where a private investor can influence the a priori distribution of stock prices by investing time in stock analysis. The alternative use of time (i.e., instead of learning about stock prices) involves working longer hours than contractually required so as to earn bonus payments.

Based on this framework, the following results are obtained. Time constraints introduce investor-specific components into the structure of optimal portfolio holdings. Moreover, time constraints make it optimal for decision makers to neither analyze one stock completely nor to invest an equal amount of time in the analysis of each stock. Therefore, decision makers have different information on different stocks at different points of calendar time even though the amount of publicly available data has not changed. Consequently, as it is reasonable to adapt the portfolio strategy to this unequal level of information, insufficient diversification and frequent portfolio restructuring can be seen as rational behavior.

To better illustrate the contributions of our paper, we contrast it with the literature. Our paper's normative portfolio model with time constraints distinguishes itself from the literature on learning constraints (van Nieuwerburgh/Veldkamp, 2005; Peng, 2005) in three major aspects. First, this literature constrains learning by use of an entropy constraint following Sims (2003); we use time constraints instead. Since entropy constraints are often justified based on limited computer capacity and computer capacity might be increased via investment in IT technology, entropy constraints can be approximated via wealth constraints. However, decision makers can do little to increase limited time (see, e.g., Mintzberg, 1973, p. 173) and thus time constraints remain a problem even if decision makers are not confronted with strictly binding budget constraints. Second, van Nieuwerburgh/Veldkamp (2005) and Peng (2005) work with model-exogenous learning constraints whereas our model employs endogenous learning constraints: Time must be optimally divided between either stock analysis or extra work that will earn bonus payments, thus making the time budget for stock analysis endogenous. Third, van Nieuwerburgh/Veldkamp (2005) and Peng (2005) deal only with insufficient diversification, but neglect excessive trading; we look at both.

Our paper's normative portfolio model is also different from Ahn/Kim/Yoon's (2006) portfolio selection with time constraints. They use a model-exogenous time constraint to penalize holdings of the risky asset, similar to transaction costs, and thereby explain investors' limited participation in the stock market. However, their time constraint is not designed to cope with the influence of time constraints on investors' learning. In particular, Ahn/Kim/Yoon (2006) neither derive the optimal time allocation between several stocks nor do they analyze the interactions between portfolio selection and time allocation.

Finally, our paper's application aspect, the use of normative portfolio selection with time constraints to explain the real-world phenomena of insufficient diversification and excessive trading, distinguishes it from that part of behavioral finance literature that deals with portfolio selection (see, e.g., Barberis/Thaler, 2003). As opposed to behavioral finance, which uses relatively frictionless markets and bounded rational investors to explain insufficient diversification and excessive trading, this paper employs learning constraints in the form of time constraints and fully rational investors.

The paper is organized as follows. The remainder of Section 1 outlines the model setup. In Section 2, the optimal solution to portfolio selection and time allocation is derived. Section 3 applies the normative model of Section 2 to insufficient diversification and excessive trading. The paper ends with a conclusion (Section 4) and a formal appendix.

1.2. Model setup

Two forces drive the selection of our model setup. First, time constraints must be adequately portrayed; second, explicit solutions for investment decisions should be obtainable to enable economic interpretations.

An adequate representation of time constraints calls for a discrete-time model. In a continuous-time model such as Peng (2005), a time constraint cannot be integrated since the learning process must be instantaneous by definition. A lower speed of learning can be captured only with discrete-time models. Unfortunately, discrete-time models often cannot be solved in explicit form (e.g., Breeden, 2004), so in this respect continuous-time models are preferable because they yield an easy-to-handle μ-σ-calculus. To deal with these conflicting requirements, we chose a compromise framework that is outlined by the following assumptions.

Assumption 1: Objective function of the decision maker

Our decision maker is a private investor who does not work as a professional portfolio manager. Otherwise, decisions about how much time to spend on work as opposed to investment analysis would not be relevant. The private investor has exponential utility and maximizes expected utility over terminal wealth. His objective function reads:

$$\max \; E\left\{ -\frac{1}{\alpha} \cdot e^{-\alpha \cdot W_T} \right\} \tag{1}$$

where α denotes the private investor's absolute risk aversion, $E\{\}$ the unconditional expectation operator, and W_T is terminal wealth at planning horizon T.

Assumption 2: Investment opportunity set

The private investor can choose between n risky stocks and one riskless asset. Stocks are not subject to short selling constraints, and their prices are jointly normally distributed. The riskless rate is constant through time, identical for borrowing and lending, and the term structure is assumed to be flat.

In addition to stochastic income from capital investments, the private investor receives deterministic income from employment. This income consists of two parts: contractual income from employment and income from bonus payments. The contractual income from employment is independent of additional working hours and by definition fixed. Bonus payments equal zero if no additional hours are spent working, but are some positive amount when extra hours are worked. Bonus payments can reach a maximum since companies do not usually offer indefinitely high bonus payments. Bonus payments are a concave function of additional working hours due to assumed decreasing marginal labor productivity.

Finally, to simplify notation, we assume that the length of the time interval during which the private investor cannot rebalance his portfolio holdings is the same as the time interval that is the basis for the determination of bonus payments. For example, if private investors receive monthly bonus payments, they rebalance portfolios on a monthly basis as well.

Assumption 3: Learning process

It is assumed that all investors have the same free access to data such as historical stock quotes, companies' fundamental data, and analysts' reports and that no investor is privy to insider information.

Historical stock quotes allow deriving a priori distributions of stock prices. Stock quotes, companies' fundamental data, and analysts' reports can be regarded as signals from which investors derive a posteriori distributions. If a posteriori distributions are more informative than a priori distributions, information is different from data; otherwise, data and information are equal. The transition from a priori to a posteriori distributions constitutes the learning process by which private investors can influence how much the a posteriori distribution is more informative than the a priori distribution by investing time. Note that investment in the riskless asset does not require learning.

More formally, the learning process develops as follows. It is assumed that signals and stock prices are jointly normally distributed. This means that the a posteriori distribution is normally distributed and can be characterized completely via the vector of conditional means and the conditional variance/covariance matrix. Furthermore, both the conditional mean $\mathrm{E}\{\mathbf{P}_\mathrm{T}|\mathbf{S}\}$ and the conditional variance/covariance matrix $\mathbf{C}_{\mathrm{P_T|S}}$ are functions of correlation coefficients between the random variables signals and stock prices (see, e.g., Mardia/Kent/Bibby, 1992, p. 63)

$$\mathrm{E}\{\mathbf{P}_\mathrm{T}|\mathbf{S}\} = \mathrm{E}\{\mathbf{P}_\mathrm{T}\} + \mathbf{COV}_{\mathrm{P_TS}}\,\mathbf{C}_\mathrm{S}^{-1}(\mathbf{S} - \mathrm{E}\{\mathbf{S}\}) \tag{2}$$

$$\mathbf{C}_{\mathrm{P_T|S}} = \mathbf{C}_{\mathrm{P_T}} - \mathbf{COV}_{\mathrm{P_TS}}\,\mathbf{C}_\mathrm{S}^{-1}\mathbf{COV}'_{\mathrm{P_TS}} \tag{3}$$

where $\mathbf{S}$ denotes the $\mathrm{m} \times 1$ vector of signals, $\mathrm{E}\{\mathbf{S}\}$ the $\mathrm{m} \times 1$ vector of unconditional expected values of signals, $\mathrm{E}\{\mathbf{P}_\mathrm{T}\}$ the $\mathrm{n} \times 1$ vector of unconditional expected values of stock prices at calendar time T, and $\mathbf{COV}_{\mathrm{P_TS}}$ is the $\mathrm{n} \times \mathrm{m}$ unconditional covariance matrix between stock prices at calendar time T and signals. $\mathbf{C}_\mathrm{S}$ is the $\mathrm{m} \times \mathrm{m}$ unconditional variance/covariance matrix of signals, $\mathbf{C}_{\mathrm{P_T}}$ the $\mathrm{n} \times \mathrm{n}$ unconditional variance/covariance matrix of stock prices at calendar time T.

Therefore, in our model, learning means that the private investor improves the correlation coefficient between signals and stock prices via time investment $\mathbf{t}$ and it holds $\mathbf{COV}_{\mathrm{P_TS}}(\mathbf{t})$ instead of $\mathbf{COV}_{\mathrm{P_TS}}$ as in Equations (2) and (3). If no time is invested, correlation coefficients between signals and stock prices equal zero and a priori and a posteriori distributions coincide. The more time the private investor invests in stock analysis, the closer the absolute values of correlation coefficients converge toward 1 and the more informative a posteriori distributions of stock prices become. This increase in absolute values of the correlation coefficients is concave in the time invested because it seems reasonable to assume that learning exhibits decreasing marginal productivity. Keep in mind that once the a posteriori distribution has been derived from stock analysis, this knowledge can be applied to any numbers of stocks bought and sold. For example, to obtain information on 10 pieces of stock i, the same amount of time must be invested as for obtaining information on one piece of stock i.

Finally, it is assumed that the private investor is "small" in the sense that his transactions do not influence stock prices. Therefore, stock prices do not reflect the information via learning gleaned by the private investor.

Assumption 4: Time constraint

The private investor must meet his physiological needs and work the contractually required number of hours. He wants to spend any additional time available either working more hours so as to earn a bonus and/or learning about stocks. In summary, although the private investor is rational, he is subject to learning constraints in the form of the following time constraint:

$$\overline{T}_\theta = \sum_{j=1}^{m} \sum_{i=1}^{n} t_{i,S_j,\theta} + t_{h,\theta} \qquad (4)$$

where $\overline{T}_\theta$ denotes the time available for stock analysis and acquiring bonus payments at calendar time $\theta \in$ {current calendar time τ, $\tau + 1$, …, $T - 1$, planning horizon T), $t_{i,S_j,\theta}$ the time invested in the analysis of signal S_j (out of m signals) for stock i (out of n stocks) at calendar time θ, and $t_{h,\theta}$ is the time invested in acquiring bonus payments via working longer than contractually required at calendar time θ.

Two things must be kept in mind when considering Equation (4). First, $\overline{T}_\theta$ is the time available after time for physiological needs (e.g., eating, sleeping, etc.) and contractual working hours are deducted from total time available. Total time available equals the length of the time interval in our model as outlined in Assumption 2. Second, $\overline{T}_\theta$ is investor-specific; e.g., a private investor contractually required to work eight hours per day will have less time for stock analysis and fewer hours available in which to earn bonus payments than a private investor who is contractually required to work only six hours per day.

2. Portfolio selection and time allocation

To analyze optimal portfolio selection and time allocation, we proceed in two steps. First, we develop the general portfolio and time allocation model. Second, we discuss special cases to illustrate optimal portfolio and time allocation since the general portfolio and time allocation model does not have closed-form solutions.

2.1. General portfolio selection and time allocation

Intuitively, the process of portfolio and time allocation evolves as follows. At current calendar time τ the private investor chooses, first, the time to be invested in the analysis of each stock. Then he observes a realization of each signal. Second, based on the realization of each signal, he selects his portfolio of stocks. At calendar time $\tau + 1$, he obtains wealth as a consequence of his portfolio decision at calendar time τ. Using wealth at calendar time $\tau + 1$ as the starting point, the process of portfolio selection with time constraints starts anew – the private investor determines his time allocation based on the wealth level at calendar time $\tau + 1$, observes new signals at calendar time $\tau + 1$, and may even be able to use the signals observed at calendar time τ in the form of intertemporal learning. In accordance with the signals observed at calendar times τ and $\tau + 1$ and the wealth level achieved at calendar time $\tau + 1$, he selects his portfolio holdings. This process is repeated every calendar time θ until calendar time $T – 1$.

To implement this process, the private investor uses backward induction. He first derives portfolio holdings at calendar time $T – 1$ for every possible realization of wealth at calendar time $T – 1$ and every possible realization of signals between calendar times τ and $T – 1$. Based on this optimal conditional portfolio selection, the private investor next determines the optimal time allocation at calendar time $T – 1$. Using optimal conditional portfolio selection and time allocation at calendar time T – 1, he calculates optimal portfolio selection at calendar time T – 2 for every possible realization of signals between calendar times τ and T – 2. These portfolio holdings are the starting point for determining the optimal time allocation at calendar time T – 2. This process is repeated until calendar time τ. Formally, the decision problem reads as follows:

$$\underbrace{\underset{\mathbf{t}_\tau}{\mathrm{Max}}\,E\left\{\underbrace{\underset{\mathbf{N}_\tau}{\mathrm{Max}}\,E\cdots\left\{\underbrace{\underset{\mathbf{t}_{T-1}}{\mathrm{Max}}\,E\left\{\underbrace{\underset{\mathbf{N}_{T-1}}{\mathrm{Max}}\,E\left\{-\frac{1}{\alpha}\cdot e^{-\alpha\cdot W_T}\middle|\mathbf{S}_{T-1},\mathbf{S}_{\tau,T-2},W_{T-1}\right\}}_{\text{inner problem at time } T-1}\middle|\mathbf{S}_{\tau,T-2},W_{T-1}\right\}}_{\text{outer problem at time } T-1}\right\}\cdots}_{\text{inner problem at time } \tau}\middle|\mathbf{S}_\tau\right\}}_{\text{outer problem at time } \tau} \tag{5}$$

s.t.: $\overline{T}_\theta = \sum_{j=1}^{m}\sum_{i=1}^{n} t_{i,S_j,\theta} + t_{h,\theta}$ for all $\theta \in \{\tau, \tau + 1, \ldots, T - 1\}$

$t_{i,S_j,\theta} \geq 0$ for all $i \in \{1,\ldots,n\}$ and $j \in \{1,\ldots,m\}$
for all $\theta \in \{\tau, \tau + 1, \ldots, T - 1\}$

$0 \leq t_{h,\theta} \leq \overline{T}_{h,\theta}$ for all $\theta \in \{\tau, \tau + 1, \ldots, T - 1\}$

with: $W_{\theta+1} = \mathbf{N}'_\theta\left(\mathbf{P}_{\theta+1} - (1+r)\cdot\mathbf{P}_\theta\right) + h\left(t_{h,\theta}\right) + W_\theta\cdot(1+r)$

where $\mathbf{N}_\theta$ denotes the n × 1 vector of numbers of stocks bought or sold at calendar time θ, W_θ wealth at calendar time θ, ` transposition of vectors or matrices, $\mathbf{P}_{\theta+1}$ the n × 1 vector of stock prices at calendar time θ + 1, r the riskless rate, $h(t_{h,\theta})$ deterministic bonus payments at calendar time θ + 1 as a function of additional working hours at calendar time θ, and $\overline{T}_{h,\theta}$ is the time investment at calendar time θ that leads to maximum bonus payments. $\mathbf{S}_{T-1}$ is the m × 1 vector of signals for all stocks at calendar time T – 1, $\mathbf{S}_{\tau,T-2}$ stands for signals for all stocks and at all calendar times between τ and T – 2 (encompasses intertemporal learning), and $\mathbf{t}_\theta$ is the vector of time invested in the analysis of all signals for all stocks at calendar time θ.

In the general model, time constraints are subject to an assumption concerning stock price movements during the process of stock analysis. Since learning does not happen instantaneously but, by definition, takes time, stock prices will change between the beginning and end of the stock analysis period, i.e., during the learning process. Additionally, a new stock price might contain new information, meaning that the learning process must begin anew, meaning that more time will pass, during which, possibly, the stock will change price again, containing yet more new information etc. To avoid this circular path, we assume, in our model, that stock analysis happens outside of trading hours. This assumption solves the circular-path problem because stock prices will no longer change during the process of stock analysis. We believe this to be a reasonable assumption since private investors will usually be at their regular employment during stock trading hours.

The solution to decision problem (5) is as follows. The optimal portfolio holdings at calendar time T – 1 stem from the solution to the inner problem at calendar time T – 1: $\mathbf{N}_{T-1}(\mathbf{S}_{\tau,T-2}, \mathbf{S}_{T-1}, W_{T-1})$. Therefore, portfolio holdings at calendar time T – 1 are conditional on W_{T-1}, $\mathbf{S}_{\tau,T-2}$, and $\mathbf{S}_{T-1}$, as well as on the time invested in stock analysis at all calendar times between τ and T – 1. Therefore, the dependence of $\mathbf{N}_{T-1}(\mathbf{S}_{\tau,T-2}, \mathbf{S}_{T-1}, W_{T-1})$ on $\mathbf{S}_{\tau,T-2}$ indicates intertemporal learning. The optimal time investment in stock analysis at calendar time T – 1 can be derived from the solution to the outer problem at calendar time T – 1: $\mathbf{t}_{T-1}(\mathbf{S}_{\tau,T-2}, W_{T-1})$. This makes the optimal time investment at calendar time T – 1 conditional on W_{T-1} and $\mathbf{S}_{\tau,T-2}$ and, as such, conditional on the time invested at all calendar times between τ and T – 2. The solution to the inner problem at calendar time T – 2 yields optimal portfolio holdings at calendar time T – 2: $\mathbf{N}_{T-2}(\mathbf{S}_{\tau,T-3}, \mathbf{S}_{T-2}, W_{T-2})$ that are conditional on both $\mathbf{S}_{\tau,T-2}$ and the time invested at each calendar time between τ and T – 2. Finally, the optimal time investment at calendar time T – 2 ($\mathbf{t}_{T-2}$) stems from the solution to the outer problem at calendar time T – 2.

This process of determining optimal portfolio holdings and time allocations is repeated until calendar time τ.

Obviously, decision problem (5) is impossible to solve in its general form.[2] On the one hand, the optimal time investment cannot be derived in explicit form since learning is nonlinear due to decreasing marginal productivity (see Assumption 3). On the other hand, conditional expectations contain optimal portfolio holdings and time investments and, as such, are highly nonlinear functions of the random variables stock prices and signals. Therefore, the repeated calculation of conditional expectations for calendar times T, T – 1, T – 2,…, τ is beyond an explicit solution.

Consequently, we analyze intertemporal learning in more detail instead of deriving the formal characteristics of optimal portfolio holdings and time allocations in the general case. The forms of intertemporal learning are:

1. The correlation between $\mathbf{P}_{\theta+2}$ and $\mathbf{S}_{\tau,\theta}$ allows the private investor to exert direct influence on $\boldsymbol{\rho}_{P_{\theta+2}S_{\tau,\theta}}(\mathbf{t}_\theta)$ via time investment, i.e., the a posteriori distribution of $\mathbf{P}_{\theta+2}$ can be improved through signals that have occurred at least two periods earlier. This form of learning, however, stresses the time constraint at calendar time θ.

2. The private investor may learn about correlation coefficients between signals and stock prices in the form of $\boldsymbol{\rho}_{P_{\theta+1}S_\theta}(\mathbf{t}_{\theta-1}+\mathbf{t}_\theta)$. This means that the private investor does not completely forget what he learned in previous periods about the connection between stock prices and signals and thus learning becomes easier at later calendar times, here termed “intertemporal informational synergies.”

3. A correlation between signals $\mathbf{S}_{\tau,\theta}$ and $\mathbf{S}_{\theta+1}$ makes the a posteriori distribution of $\mathbf{P}_{\theta+2}$ more informative than its a priori distribution.

[2] Appendix 2 contains some calculations to illustrate the solutions to the special case of a two-period problem.

4. A correlation between $\mathbf{P}_{\theta+1}$ und $\mathbf{P}_{\theta+2}$ also improves the a posterior distribution of $\mathbf{P}_{\theta+2}$.

The second, third, and fourth forms of learning do not stress time constraints at calendar time θ.

The fact that information about individual stocks can change due to intertemporal learning, even though the amount of data has not necessarily changed, has an interesting consequence for portfolio selection. The private investor must update his portfolio via trading to take advantage of the new information.

2.2. Special cases of portfolio selection and time allocation

Since decision problem (5) cannot be solved in explicit form, it is difficult to gain an adequate understanding of optimal portfolio holdings and time allocations with time constraints. Thus we next consider special cases that bring us closer to or even achieve explicit solutions of the optimal portfolio and time allocation problem.

2.2.1. First special case: Portfolio selection and time allocation in the last period

In the first special case it is assumed that the private investor has reached calendar time $\tau = T - 1$ so that he is just one period prior to his planning horizon T. This means that general decision problem (5) simplifies to:[3]

$$\underbrace{\underset{\mathbf{t}_{T-1}}{\text{Max}}\, E\left\{ \underbrace{\underset{\mathbf{N}_{T-1}}{\text{Max}}\, E\left\{ -\frac{1}{\alpha} \cdot e^{-\alpha \cdot W_T} \middle| \mathbf{S}_{T-1} \right\}}_{\text{inner problem at } T-1} \right\}}_{\text{outer problem at } T-1} \tag{6}$$

$$\text{s.t.:} \quad \overline{T}_{T-1} = \sum_{j=1}^{m} \sum_{i=1}^{n} t_{i,S_j,T-1} + t_{h,T-1}$$

$$t_{i,S_j,T-1} \geq 0 \qquad \text{for all } i \in \{1,\ldots,n\} \text{ and } j \in \{1,\ldots,m\}$$

[3] Note that this special case is not identical to focusing on the period between T – 1 and T of decision problem (5). In the latter case, optimal portfolio holdings and time allocations at calendar time T – 1 are conditional on

$$0 \le t_{h,T-1} \le \overline{T}_{h,T-1}$$

with: $W_T = \mathbf{N}'_{T-1}\left(\mathbf{P}_T - (1+r)\cdot\mathbf{P}_{T-1}\right) + W_{T-1}\cdot(1+r) + h\left(t_{h,T-1}\right)$

Optimal portfolio holdings follow from the solution to the inner problem, which solution can be found by solving the following equivalent problem:[4]

$$\underset{\mathbf{N}_{T-1}}{\text{Max}}\ E\{W_T|\mathbf{S}\} - \frac{1}{2}\alpha\cdot\text{var}\left(W_T|\mathbf{S}\right) \tag{7}$$

with: $E\{W_T|\mathbf{S}\} = \mathbf{N}'_{T-1}\left(E\{\mathbf{P}_T|\mathbf{S}\} - (1+r)\cdot\mathbf{P}_{T-1}\right) + W_{T-1}\cdot(1+r) + h\left(t_{h,T-1}\right)$

$$\text{var}\left(W_T|\mathbf{S}\right) = \mathbf{N}'_{T-1}\mathbf{C}_{P_T|S}(\mathbf{t}_{T-1})\,\mathbf{N}_{T-1}$$

Relying on the definitions of conditional expectations (Equation (2)) and variance/covariances matrices (Equation (3)), the following optimal portfolio holdings are obtained as the solution to decision problem (7):

$$\mathbf{N}_{T-1} = \frac{1}{\alpha}\left[\mathbf{C}_{P_T} - \mathbf{COV}_{P_TS}(\mathbf{t}_{T-1})\mathbf{C}_S^{-1}\mathbf{COV}'_{P_TS}(\mathbf{t}_{T-1})\right]^{-1}\left(E\{\mathbf{P}_T\} - (1+r)\cdot\mathbf{P}_{T-1}\right) \tag{8}$$

$$+\frac{1}{\alpha}\left[\mathbf{C}_{P_T} - \mathbf{COV}_{P_TS}(\mathbf{t}_{T-1})\mathbf{C}_S^{-1}\mathbf{COV}'_{P_TS}(\mathbf{t}_{T-1})\right]^{-1}\mathbf{COV}_{P_TS}(\mathbf{t}_{T-1})\mathbf{C}_S^{-1}\left(\mathbf{S} - E\{\mathbf{S}\}\right)$$

The portfolio holdings (8) consist of three components. First, a volume component $\frac{1}{\alpha}$ that determines the allocation of funds between risky and riskless assets. Second, a structural component that allocates the risky invested funds to single stocks. This structural component itself consists of two parts. The first part (first line of Equation (8)), is the tradeoff between expected value and risk of each stock that can be influenced through learning $\mathbf{COV}_{P_TS}(\mathbf{t}_{T-1})$. The second part (second line of Equation (8)) is composed of a correction portfolio that adapts portfolio holdings to signal observations. Note that risk $\mathbf{COV}_{P_TS}(\mathbf{t}_{T-1})$ dependent on learning

the optimal portfolio and time allocation decisions at all calendar times prior to T – 1. In special case (6), the one-period decision is, by definition, unconditional on all calendar times prior to T – 1.

4 Since signals occur only at calendar time T – 1, we henceforth drop the signals' time index to simplify notation.

and the correction portfolio are exactly those components that distinguish the portfolio holdings (8) from neoclassical optimal portfolio holdings in the hybrid model, i.e.,

$$\mathbf{N}_{T-1} = \frac{1}{\alpha}\mathbf{C}_{P_T}^{-1}\left(E\{\mathbf{P}_T\} - (1+r)\cdot\mathbf{P}_{T-1}\right) \tag{9}$$

The decomposition of portfolio holdings into an investor-dependent volume component and an investor-independent structural component is known as Tobin separation. Equation (8) shows that the Tobin separation breaks down in the event of time constraints. The structural component contains learning-dependent risk $\mathbf{COV}_{P_T S}(\mathbf{t}_{T-1})$ as a function of time invested in stock analysis. The time invested in stock analysis, however, is investor-specific because it depends on both the investor-specific speed of learning and the time constraint.

Inserting the optimal portfolio holdings (8) into the inner decision problem (6) provides the foundation for calculating optimal time allocations. In other words, the outer problem of decision problem (6) reads (see Appendix A.1):

$$\underset{\mathbf{t}_{T-1}}{\text{Min}}\ e^{-\alpha\cdot W_{T-1}\cdot(1+r)-\alpha\cdot h(t_{h,T-1})}\cdot e^{-\frac{1}{2}\cdot E\{\mathbf{P}_T-(1+r)\cdot\mathbf{P}_{T-1}\}'\mathbf{C}_{P_T}^{-1}E\{\mathbf{P}_T-(1+r)\cdot\mathbf{P}_{T-1}\}}\cdot\sqrt{\det\left(\mathbf{B}^{-1}(\mathbf{t}_{T-1})\right)} \tag{10}$$

s.t.: $\overline{T}_{T-1} = \sum_{j=1}^{m}\sum_{i=1}^{n} t_{i,S_j,T-1} + t_{h,T-1}$

$t_{i,S_j,T-1} \geq 0$ for all $i \in \{1,\dots,n\}$ and $j \in \{1,\dots,m\}$

$0 \leq t_{h,T-1} \leq \overline{T}_{h,T-1}$

with: $\mathbf{B}(\mathbf{t}_{T-1}) \equiv \text{Id}$

$$+\left(\text{cholesky}\left(\boldsymbol{\rho}_{SS}^{-1}\right)\right)'\boldsymbol{\rho}'_{P_T S}(\mathbf{t}_{T-1})\left[\boldsymbol{\rho}_{P_T P_T} - \boldsymbol{\rho}_{P_T S}(\mathbf{t}_{T-1})\boldsymbol{\rho}_{SS}^{-1}\boldsymbol{\rho}'_{P_T S}(\mathbf{t}_{T-1})\right]^{-1}\boldsymbol{\rho}_{P_T S}(\mathbf{t}_{T-1})\,\text{cholesky}\left(\boldsymbol{\rho}_{SS}^{-1}\right)$$

where det(.) denotes the determinant of a matrix, Id denotes the $m \times m$ identity matrix, $\boldsymbol{\rho}_{SS}$ the $m \times m$ matrix of correlation coefficients between signals, $\boldsymbol{\rho}_{P_T P_T}$ the $n \times n$ matrix of correlation coefficients between stock prices at calendar time T, and $\boldsymbol{\rho}_{P_T S}(\mathbf{t}_{T-1})$ is the $n \times m$ matrix of correlation coefficients between stock prices at calendar time T and signals. Note the dependence of $\boldsymbol{\rho}_{P_T S}(\mathbf{t}_{T-1})$ on the time invested in stock analysis and, thus, the potential to improve stock analysis through investing time.

The necessary condition of the time ($t_{i,S_j,T-1}$) invested in stock i's analysis through learning about its connection to signal S_j reads (interior solution):

$$\underbrace{-\alpha \frac{d\,h\left(t_{h,T-1}\right)}{d\,t_{i,S_j,T-1}}}_{\text{"income effect"}} + \underbrace{\frac{1}{2} \cdot \frac{\frac{d\det\left(\mathbf{B}^{-1}(\mathbf{t}_{T-1})\right)}{d\,t_{i,S_j,T-1}}}{\det\left(\mathbf{B}^{-1}(\mathbf{t}_{T-1})\right)}}_{\text{"distributional effect"}} = 0 \tag{11}$$

According to Equation (11), the optimum time allocation is determined in a two-step procedure. Note that in actuality, both steps occur simultaneously and are separated here for illustrative purposes only.

In the first step, it is determined how the time budget is divided between learning about stocks on the one hand, and working extra hours to earn bonus payments on the other hand. In the optimum, the negative impact of investing time in stock analysis on riskless bonus payments ("income effect") must be exactly offset by its positive effect on stocks' a posteriori distributions ("distributional effect"). The "income effect" stems from the fact that a higher time investment in stock analysis leads to a decrease in riskless bonus payments because time invested in stock analysis cannot be used to earn bonus payments by working extra hours. Note, however, that both effects have different starting points. The "income effect" describes direct, the "distributional effect" indirect consequences of learning on the private investor's objectives. The indirect consequences stem from the fact that the "distributional effect" needs a transformation vehicle, namely, optimal portfolio holdings $\mathbf{N}_{T-1}$, to enter the private investor's objectives. Furthermore, stock prices at calendar time T are random variables and a better a posteriori distribution is no guarantee that the private investor achieves higher utility ex post. Consequently, a private investor with a higher absolute risk aversion α invests more time in stock analysis to improve correlation coefficients between signals and stock prices. The "distributional effect" is especially pronounced in the event of informational synergies. "Informational synergies" occur when information about several stocks can be obtained by analyzing just one stock. More formally, the time invested in analysis of signal S_j exerts influence on the correlation coefficients of stocks i and i + 1, e.g., $\rho_{P_{i,T}S_j}\left(t_{j,T-1}\right)$ and $\rho_{P_{i+1,T}S_j}\left(t_{j,T-1}\right)$, compared to the absence of informational synergies where $\rho_{P_{i,T}S_j}\left(t_{i,S_j,T-1}\right)$ and $\rho_{P_{i+1,T}S_j}\left(t_{i+1,S_j,T-1}\right)$ holds.

The second step involves dividing the time budget for stock analysis as a whole, determined in the first step, between individual stocks.

2.2.2. Second special case: Portfolio selection and time allocation in the last period with specified learning and bonus payment functions

To characterize the optimal time allocation further and, in particular, to examine the "distributional effect" beyond the general statements made in Section 2.2.1, it is necessary to solve Equation (11). This task can be achieved only by particularizing the bonus payment and learning functions.

In a first step, assume $h(t_{h,T-1}) \equiv \frac{W_{max}}{\overline{\overline{T}}_{h,T-1}} \cdot t_{h,T-1} = \frac{W_{max}}{\overline{\overline{T}}_{h,T-1}} \cdot \left(\overline{T}_{T-1} - \sum_{j=1}^{m}\sum_{i=1}^{n} t_{i,S_j,T-1} \right)$, i.e., a linear bonus payment function. Then, Equation (11) simplifies to

$$\alpha \underbrace{\frac{W_{max}}{\overline{\overline{T}}_{h,T-1}}}_{\text{"income effect"}} + \frac{1}{2} \cdot \underbrace{\frac{\frac{d\,\det\left(\mathbf{B}^{-1}(\mathbf{t}_{t-1})\right)}{d\,t_{i,S_j,T-1}}}{\det\left(\mathbf{B}^{-1}(\mathbf{t}_{T-1})\right)}}_{\text{"distributional effect"}} = 0 \tag{12}$$

Equation (12) shows that both the optimal time investment in the acquisition of bonus payments and in the analysis of signal S_j of stock i is independent of $\overline{T}_{T-1}$. Obviously, a private investor with a high time budget due to, e.g., a low number of contractual working hours, will choose an allocation of time between the acquisition of bonus payments and stock analysis that is identical to that chosen by a private investor with a low time budget. This is because a linear bonus payment function means that the “income effect” is independent of $t_{h,T-1}$, and, thus, via the time constraint in problem (10), independent of $\overline{T}_{T-1}$. This constant “income effect” is complemented by a “distributional effect” that depends by definition only on $t_{i,S_j,T-1}$, not on $\overline{T}_{T-1}$.

In a second step, assume uncorrelated signals and uncorrelated stocks in addition to linear bonus payment functions. Furthermore, specify a learning environment where one stock i has only one signal S_i, there are no informational synergies, and learning in the form of stock analysis develops according to

$$\rho_{P_{i,T}S_i}(t_{i,S_i,T-1}) = \sqrt{\frac{t_{i,S_i,T-1}}{X_{i,S_i,T-1}}} \qquad (13)$$

where $X_{i,S_i,T-1}$ is the time that must be invested in analysis of signal S_i of stock i so that the correlation coefficient between signal S_i and stock i’s price equals 1.

The higher $X_{i,S_i,T-1}$ is, the more data are available on stock i and the higher the time investment must be to reach a certain correlation coefficient between stock prices and signals compared to a lower $X_{i,S_i,T-1}$. Therefore, it is reasonable to set $X_{i,S_i,T-1}$ larger than $\overline{T}_{T-1}$ because then correlation coefficients between stock prices and signals cannot reach 1, i.e., stocks cannot be analyzed completely. Despite the dependence of $X_{i,S_i,T-1}$ on stock i, signal S_i, and calendar time, $X_{i,S_i,T-1}$ is independent of the individual private investor, for $X_{i,S_i,T-1}$ is related to data and the amount of data is identical for all investors according to Assumption 3.

Individual aspects do affect learning however, according to (13), through the speed of learning. A private investor with a learning function according to Equation (13) takes $t_{i,S_i,T-1} = \overline{\rho}^2_{P_{i,T}S_i} \cdot X_{i,S_i,T-1}$ to reach a certain correlation level $\overline{\rho}_{P_{i,T}S_i}$; a private investor with a learning function $\rho_{P_{i,T}S_i}(t_{i,S_i,T-1}) = \frac{t_{i,S_i,T-1}}{X_{i,S_i,T-1}}$ takes longer, namely, $t_{i,S_i,T-1} = \overline{\rho}_{P_{i,T}S_i} \cdot X_{i,S_i,T-1}$.

Based on the learning function of Equation (13), the optimal time investment $t_{i,S_i,t-1}$ is (see Appendix A.1.4):

$$t_{i,S_i,t-1} = X_{i,S_i,T-1} - \frac{1}{2} \cdot \frac{\overline{T}_{h,T-1}}{\alpha \cdot W_{max}} \tag{14}$$

Equation (14) provides several insights into optimal time allocations. First, the private investor does not analyze one stock completely. This is because he holds a portfolio of stocks and wants to learn something about each stock in the portfolio. This is especially true as there are no informational synergies in the sense that information about all stocks cannot be obtained by analyzing any one stock.[5]

Second, the private investor does not spend an equal amount of time analyzing each stock. Instead, he invests more time analyzing those stocks for which more data are available (stocks with higher $X_{i,S_i,T-1}$). Stocks for which less data are available (stocks with lower $X_{i,S_i,T-1}$) do not need as high a time investment to achieve an adequate $\rho_{P_{i,T}S_i}(t_{i,S_i,T-1})$ as do stocks for which more data are available. To get a feeling which types of stocks have a high and which have a low $X_{i,S_i,T-1}$, consider real-world stock analysis. Smallcap and midcap companies, which have great difficulty in attracting analyst coverage (see, e.g., Shearer, 2003, p. 2), create less data than large companies or exciting high-growth companies. Less data result in a smaller $X_{i,S_i,T-1}$ for smallcap and midcap companies: $X_{small\,cap,S_{small\,cap},T-1} < X_{large\,cap,S_{large\,cap},T-1}$. Moreover, complex signals like balance sheets are more difficult to analyze than simpler signals like order flow of a company; therefore, $X_{i,balance\,sheet_i,T-1} > X_{i,order\,flow_i,T-1}$. Finally, the amount of data available about stocks can change over time. For example, in the fourth quarter of 2005, solar energy stocks received a great deal of coverage by analysts, which created a huge amount of data that had to

5 This type of learning behavior is in contrast to the one in van Nieuwerburgh/Veldkamp (2005) where investors choose to learn about one stock. The difference arises because in van Nieuwerburgh/Veldkamp (2005), stock prices have common factors that can be learned by analyzing any stock – what we call informational synergies – and, also, their investors cannot learn about stocks' risk, but only about stocks' means.

be transformed into information. Therefore, solar energy stocks changed from being quick to analyze stocks before the fourth quarter 2005 to being more slowly to analyze stocks from the fourth quarter 2005 on, i.e., $X_{solar,S_i,3rd} < X_{solar,S_i,4th}$.

Third, Equation (14) demonstrates that time investment in the analysis of stock i increases with lower $\overline{T}_{h,T-1}$ and higher W_{max}. Since the slope of the bonus payment function $\frac{W_{max}}{\overline{T}_{h,T-1}}$ increases with lower $\overline{T}_{h,T-1}$ and higher W_{max}, it becomes easier to achieve bonus payment. Therefore, private investors feel less pressure to invest time in bonus payments and the saved time can be invested in stock analysis.

3. Time constraints and a rational explanation of insufficient diversification and excessive trading

This section deals with the second goal of the paper – the application aspect. We will demonstrate that learning constraints in the form of time constraints offer a fully rational explanation for two of the most discussed real-world investment phenomena: insufficient diversification and excessive trading. Those phenomena are to date not adequately explained by neoclassical portfolio selection (see, e.g., Barberis/Thaler, 2003, Section 7).

3.1. Insufficient diversification

Insufficient diversification is characterized by portfolio holdings that are much less diversified than recommended by normative portfolio selection models (see, e.g., Barberis/Thaler, 2003, pp. 1101). However, it is not exactly clear how one would define “much less diversified than recommended by normative portfolio selection models.” In the sections that follow, we particularize insufficient diversification and illustrate how adding time constraints to the neoclassical model of portfolio selection contributes to explaining insufficient diversification.

3.1.1. Test criterion

We define the test criterion to detect potential connections between insufficient diversification and learning constraints in the form of time constraints as follows:

the development of the quotient of neoclassical portfolio holdings for two stocks i and j compared with that of the pure learning components of portfolio holdings with time constraints.

To apply this test criterion, we have to particularize its components. In this connection, we employ the special case of Section 2.2.1. Therefore, we specify the quotient of neoclassical portfolio holdings as $\frac{N_{i,T-1,neocl}}{N_{j,T-1,neocl}}$ using portfolio holdings (9). The quotient of pure learning components of portfolio holdings with time constraints consists of the tradeoff between expected value and risk $\mathbf{COV}_{P_T S}(\mathbf{t}_{T-1})$ dependent on learning, i.e., the first part of portfolio holdings (8): $\frac{N_{i,T-1,learn}}{N_{j,T-1,learn}}$.

If a decreasing time budget $\overline{T}$ yields $\frac{N_{i,T-1,learn}}{N_{j,T-1,learn}}$ (for all stock i ≠ j) farther away from 1 than $\frac{N_{i,T-1,neocl}}{N_{j,T-1,neocl}}$ (for all stock i ≠ j), then tight time constraints can contribute to a rational explanation of insufficient diversification.

The test criterion is justified as follows. Barberis/Thaler (2003, p. 1101) associate normative portfolio models with neoclassical portfolio theory. Neoclassical (unconditional) portfolio holdings, as in Tobin (1965) and Merton (1969), do not contain a reference to learning and, thus, do not distinguish between a priori and a posteriori distributions. Therefore, they can be described with the help of the portfolio holdings (9).

Since a posteriori (conditional) portfolio holdings (8) contain learning constraints in the form of time constraints, they might be a good starting point in the comparison with unconditional portfolio holdings. However, caution is needed regarding two aspects. First, conditional portfolio holdings (8) are characterized by risk $\mathbf{COV}_{P_T S}(\mathbf{t}_{T-1})$ dependent on learning and the correction portfolio. The correction portfolio contains a combination of limited learning due to time constraints and signal-induced correction terms and therefore is a mixture of two completely different components. To analyze the relation between learning constraints in the form of time constraints and insufficient diversification, it is necessary to concentrate on pure learning effects and, thus, on the tradeoff between expected value and risk $\mathbf{COV}_{P_T S}(\mathbf{t}_{T-1})$ dependent on learning. Second, a direct comparison of the portfolio holdings (9) with the pure learning effects of Equation (8) is inadequate. Equation (9) contains an information level of zero, whereas Equation (8) is characterized by various information levels depending on the time budget $\overline{T}$. To get around this problem, it is reasonable to focus on the development of $\frac{N_{i,T-1,learn}}{N_{j,T-1,learn}}$ relative to $\frac{N_{i,T-1,neocl}}{N_{j,T-1,neocl}}$ for several time budgets. Neither $\frac{N_{i,T-1,learn}}{N_{j,T-1,learn}}$ compared to $\frac{N_{i,T-1,neocl}}{N_{j,T-1,neocl}}$ for a fixed time budget nor the size of the portfolio holdings (8) compared to that based on Equation (9) are adequate measures.

3.1.2. Results and interpretation

The connections between learning constraints in the form of time constraints and insufficient diversification can be best illustrated by means of a numerical example. To do this, we will employ the framework of Section 2.2.1 (portfolio selection and time allocation in the last period) and the learning environment of Section 2.2.2 (one stock i has only one signal S_i, there are no informational synergies, and learning in the form of stock analysis develops according to Equation (13)). To further simplify the analysis, we assume that signals are uncorrelated and that there are no payments from contractual work and no bonus payments.

The following parameters are the basis for our numerical analysis.[6] The private investor can choose between two stocks and one riskless asset, with stock prices at calendar time T – 1 $P_{1,T-1} = P_{2,T-1} = 100$, expected values $E\{P_{1,T}\} = 105$ and $E\{P_{2,T}\} = 107.5$, and variance/covariance matrix $\begin{pmatrix} var(P_{1,T}) & \sigma_{P_{1,T}} \cdot \sigma_{P_{2,T}} \cdot \rho_{P_{1,T},P_{2,T}} \\ \sigma_{P_{1,T}} \cdot \sigma_{P_{2,T}} \cdot \rho_{P_{1,T},P_{2,T}} & var(P_{2,T}) \end{pmatrix} = \begin{pmatrix} 512 & 257.6 \\ 257.6 & 1058 \end{pmatrix}$. The riskless rate equals 2% per annum, and the private investor has an exogenous income of W_{T-1} = 25,000 EUR.[7] The private investor's absolute risk aversion is[8] $\alpha = \frac{1}{17000}$.

With respect to stock analysis, two scenarios are considered. In the first scenario, there are more data available for Stock 1 than for Stock 2, i.e., $X_{1,S_1,T-1} = 1 > X_{2,S_2,T-1} = 0.8$. In the second scenario, $X_{1,S_1,T-1} = 0.64 < X_{2,S_2,T-1} = 0.8$.

Using these preliminaries, we plot the test criterion quotient = $\frac{N_{i,T-1,learn}}{N_{j,T-1,learn}}$, based on Equation (8) (conditional holding), versus $\frac{N_{i,T-1,neocl}}{N_{j,T-1,neocl}}$, based on Equation (9) (unconditional holdings), as a function of the time budget $\overline{T}$ and obtain:

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[6] We do not strive to explain portfolio holdings found in the empirical literature. In particular, we do not claim that the parameters dealing with the time constraint are empirically valid although we believe they are realistic.

[7] A riskless rate of 2% is in accordance with the current term structure of interest rates in Germany. 25,000 EUR is approximately the gross national income per capita for Germany in 2004 according to World Bank statistics.

[8] The absolute risk aversion is chosen so that the portfolio weights $w_{i,t-1} = \frac{N_{i,T-1} \cdot P_{i,T-1}}{W_{T-1}}$ do not contain a short sale of one risky or the riskless asset: $w_{1,T-1}$ = 25.13%, $w_{2,T-1}$ = 29.23%, and $w_{0,T-1}$ = 45.63%.

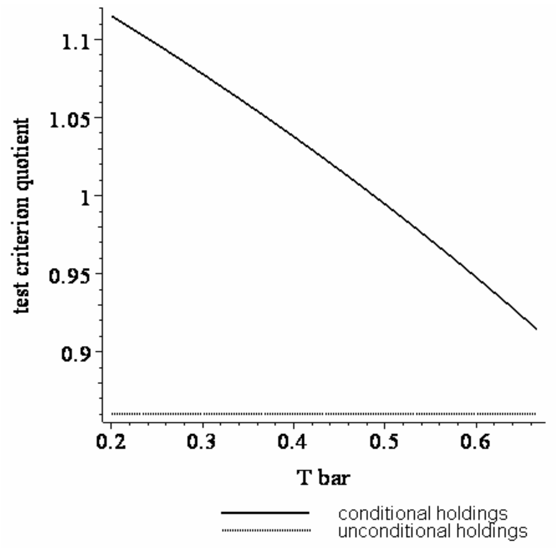

Fig. 1a. Stocks' test criterion quotients when $X_{1,S_1,T-1} = 1 > X_{2,S_2,T-1} = 0.8$ (Scenario 1)

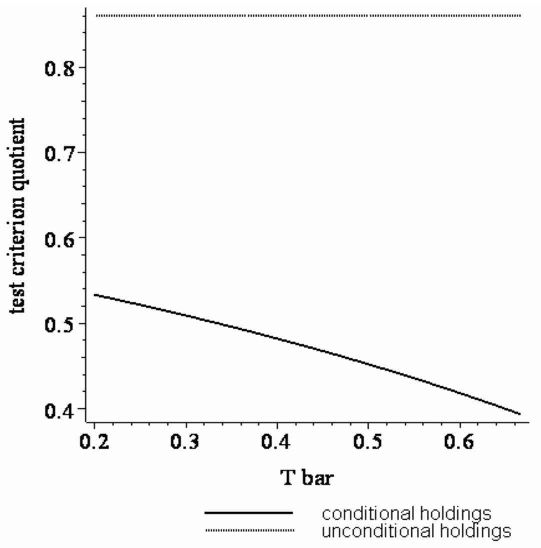

Fig. 1b. Stocks' test criterion quotients when $X_{1,S_1,T-1} = 0.64 < X_{2,S_2,T-1} = 0.8$ (Scenario 2)

Figures 1a and 1b illustrate that the interaction between availability of data (different levels of X in Scenarios 1 and 2) and time budgets ($\overline{T}$) provides rich diversification patterns, including insufficient diversification: In Scenario 1, the test criterion quotient for conditional portfolio holdings is closer to 1 than that for unconditional holdings, irrespective of the tightness of the time constraint. For time budgets $\overline{T}$ around 0.4, conditional portfolio holdings even show naïve diversification, i.e., the test criterion coefficient is around 1. By contrast, in Scenario 2, conditional portfolio holdings are significantly more unequal than unconditional portfolio holdings for all time budgets considered. This means that investors with different time budgets follow completely different levels of diversification even though they have identical data, risk aversions, and wealth. Moreover, stocks with a different amount of data (different X) induce different diversification patterns, as Scenarios 1 and 2 illustrate, although their unconditional portfolio holdings are independent of the amount of data X.

Since a private investor optimally invests a different amount of time analyzing each stock, he possesses different information on each stock in the optimum. Consequently, insufficient diversification of portfolio holdings can be explained through a normative portfolio selection model, namely, portfolio selection with learning constraints in the form of time constraints. There is no need to attribute it solely to bounded rationality.

3.2. Excessive trading

Excessive trading occurs when portfolios are restructured more often than can be justified by the availability of new information (see, e.g., Barberis/Thaler, 2003, p. 1103). However, once again, it is not exactly clear how one would define "restructured more often than can be justified by the availability of new information." In the sections that follow, we particularize excessive trading and illustrate how adding time constraints to the neoclassical model of portfolio selection contributes to explaining excessive trading.

3.2.1. Test criterion

We define the test criterion to detect potential connections between excessive trading and learning constraints in the form of time constraints as follows:

the quotient of the pure learning components of portfolio holdings with time constraints for one stock i at different calendar times T – 1 and T – 2 after the incentive to rebalance neoclassical portfolio holdings has been eliminated.

To apply this test criterion, we have to particularize its components. Based on the special case of Section 2.2.1, we specify the quotient of pure learning components of portfolio holdings with time constraints at different calendar times as $\frac{N_{i,T-1,learn}}{N_{j,T-1,learn}}$, the multi-period analogue[9] of the pure learning component of the portfolio holdings (8).

If $\frac{N_{i,T-1,learn}}{N_{j,T-1,learn}}$ (for all stocks i) differs for different time budgets $\overline{T}$ even though the incentive for rebalancing neoclassical portfolio holdings has been eliminated, then time constraints can be successfully connected with excessive trading.

The test criterion is justified as follows. The reasonableness of concentrating on the pure learning component of the portfolio holdings (8) in order to study the effects of time constraints was previously justified (see Section 3.1.1). To elaborate the frequency aspect of excessive trading it is, in addition, necessary to measure the frequency of portfolio restructurings with time constraints against the frequency of portfolio rebalancing in a neoclassical world, i.e., to separate information-induced trading from noninformation-induced trading. All neoclassical dynamic portfolio selection models advocate portfolio restructurings. For example, the discrete-time models of Fama (1970) and Hakansson (1970) restructure their optimal portfolio holdings at every point in calendar time. The continuous-time models of, e.g., Merton (1969, 1971, 1973), even rebalance portfolio holdings continuously and thus make excessive trading impossible. Portfolio rebalancing in neoclassical dynamic portfolio selection is based on the fact that calculated and actual portfolio holdings usually deviate when the random variable stock price becomes known. The reason for this noninformation-induced rebalancing is that the calculated portfolio holdings are based on moments of the stock price distribution, whereas actual portfolio holdings are based on actual stock prices. This means that neoclassical portfolio holdings are not restructured only if a certain realization of the random variable stock price occurs. This realization of the random variable stock price is what we call "compensated stock price." Using compensated stock prices and calculating portfolio holdings (8), we can be sure that every restructuring of $N_{i,T-1,learn}$ must be information induced, i.e., related to learning constraints in the form of time constraints alone.

3.2.2. Results and interpretation

The numerical analysis in this section is based on the parameters of Section 3.1.2. In addition, we use the following parameters to extend our example to the dynamic world.

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[9] A derivation of portfolio holdings for this special case is contained in Appendix A2.

The private investor is put into a two-period framework. Stock prices' expected values at calendar time T are $E\{P_{1,T}\} = 110.25$ and $E\{P_{2,T}\} = 115.56$. The variance/covariance matrix at calendar time T – 1 reads $\begin{pmatrix} 512 & 0 \\ 0 & 1058 \end{pmatrix}$, that of calendar time T $\begin{pmatrix} 1024 & 0 \\ 0 & 2116 \end{pmatrix}$, and all intertemporal correlation coefficients between stock prices are set to zero. The compensated stock prices can be calculated within this environment as follows:[10] $P_{1,T-1} = 96.23$ and $P_{2,T-1} = 91.43$. To simplify notation, we further assume $\overline{T}_{T-2} = \overline{T}_{T-1}$ and $X_{i,S_i T-2} = X_{i,S_i,T-1}$.

Using these parameters, we plot the test criterion quotient = $\frac{N_{i,T-1,learn}}{N_{j,T-1,learn}}$, based on Equation (8), versus $\frac{N_{i,T-2,neocl}}{N_{i,T-1,neocl}} = 1$, based on Equation (9), as a function of $\overline{T} = \overline{T}_{T-2} = \overline{T}_{T-1}$ and obtain:

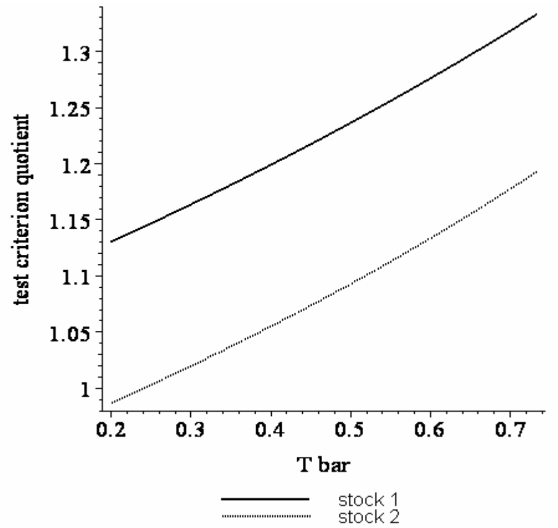

Fig. 2a. Stocks' test criterion quotients when $X_{1,S_1,T-2} = X_{1,S_1,T-1} = 1 > X_{2,S_2,T-2} = X_{2,S_2,T-1} = 0.8$ (Scenario 1)

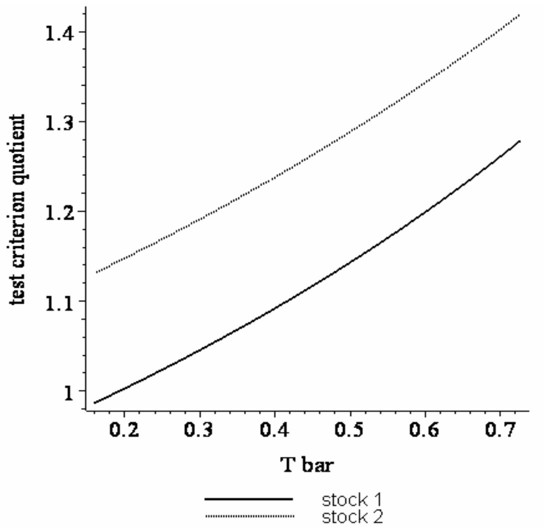

Fig. 2b. Stocks' test criterion quotients when $X_{1,S_1,T-2} = X_{1,S_1,T-1} = 0.64 < X_{2,S_2,T-2} = X_{2,S_2,T-1} = 0.8$ (Scenario 2)

Figures 2a and 2b demonstrate that different time budgets ($\overline{T}$) and different availability of data (different levels of X in Scenarios 1 and 2) lead to different portfolio restructurings since the test criterion quotients are usually unequal to 1. In fact, in this numerical example, the more time that is available, the more the private investor can learn and the more pronounced

[10] The calculations are available from the authors as Maple file.

the portfolio restructuring will be. Only for one particular $\overline{T}$ is no portfolio restructuring optimal (test criterion quotient equals 1).

This means that investors with different time budgets will restructure their portfolios differently even though they have identical data, risk aversions, and initial wealth. Moreover, stocks with different amount of data (different levels of X) induce different rebalancing patterns – see Scenarios 1 and 2 – even though their neoclassical portfolio holdings are not restructured in the optimum.

Since, at each point of calendar time, a private investor optimally spends a different amount of time analyzing each stock, he possesses different information on each stock at different calendar times. Consequently, frequent portfolio rebalancing can be explained through a normative portfolio selection model, namely, portfolio selection with learning constraints in the form of time constraints. There is no need to attribute it solely to bounded rationality or to label as "excessively" frequent portfolio rebalancing.

4. Conclusion

We began this paper with the observation that in this age of the Internet and the ready availability of financial news on television, private investors can obtain, without cost, an abundance of data concerning stocks, including historical stock quotes, companies' fundamental data, and analysts' reports. However, private investors do not have enough time to transform data into information because they must meet physiological needs and work, leaving little time to obtain information about stocks via stock analysis.

Starting from this framework, the following results are obtained. Time constraints introduce investor-specific components into the structure of portfolio holdings. Moreover, due to time constraints, it is not optimal for decision makers to either analyze one stock completely or invest an equal amount of time to the analysis of each stock. Therefore, decision makers have different information on different stocks at different calendar times even though the amount of publicly available data has not changed. Consequently, it is reasonable to adapt the portfolio strategy to this unequal level of information, which might result in insufficient diversification and frequent portfolio restructuring.

By basing our model on a fully rational instead of a bounded rational private investor, we offer a new explanation of real-world investment phenomena, phenomena that have, to date, primarily been interpreted in light of behavioral finance. We do not reject the findings based on behavioral finance; rather, we point out that there are other explanations for real-world investment phenomena. We believe we have taken a first step toward the unification of mainly descriptive behavioral finance and normative portfolio theory. Also, we believe we may have found an answer to the questions posed by Shleifer (2000, p. 195): Why do different investors have different models of what are good investments and why do they trade so much with each other? Perhaps it is because they are subject to different time constraints and, thus, have different amounts of information available to guide them.

Appendix

A.1. Optimal time allocation in the static model

To transform decision problem (6) into the basis for determining optimal time allocation in problem (10) using the optimal the portfolio holdings (8), several intermediate steps are necessary.

A.1.1. First step: Calculation of the expected value of the inner problem (6) using optimal the portfolio holdings (8)

Since W_T and signals **S** are jointly normally distributed, the expected value of the inner problem (6) reads

$$E\left\{-\frac{1}{\alpha}\cdot e^{-\alpha\cdot W_T}\middle|\mathbf{S}\right\}=-\frac{1}{\alpha}\cdot e^{-\alpha\cdot E\{W_T|\mathbf{S}\}+\frac{1}{2}\cdot\alpha^2\cdot \mathrm{var}(W_T|\mathbf{S})} \tag{A1.1}$$

with: $E\{W_T|\mathbf{S}\}=\mathbf{N}'_{T-1}\left(E\{\mathbf{P}_T|\mathbf{S}\}-(1+r)\cdot\mathbf{P}_{T-1}\right)+W_{T-1}\cdot(1+r)+h(t_{h,T-1})$

$\mathrm{var}(W_T|\mathbf{S})=\mathbf{N}'_{T-1}\mathbf{C}_{P_T|S}(\mathbf{t}_{T-1})\,\mathbf{N}_{T-1}$

Using the optimal portfolio holdings (8) – in the interests of simplification, the expression for $\mathbf{C}^{-1}_{P_T|S}(\mathbf{t}_{T-1})$ has not been substituted into portfolio holdings –

$$\mathbf{N}_{T-1}=\frac{1}{\alpha}\cdot\mathbf{C}^{-1}_{P_T|S}(\mathbf{t}_{T-1})\left(E\{\mathbf{P}_T\}-(1+r)\cdot\mathbf{P}_{T-1}\right) +\frac{1}{\alpha}\mathbf{C}^{-1}_{P_T|S}(\mathbf{t}_{T-1})\,\mathbf{COV}_{P_TS}(\mathbf{t}_{T-1})\mathbf{C}_S^{-1}(\mathbf{S}-E\{\mathbf{S}\}) \tag{A1.2}$$

$E\{W_T|\mathbf{S}\}$ and $\mathrm{var}(W_T|\mathbf{S})$ in Equation (A1.1) can be calculated.

$E\{W_T|\mathbf{S}\}$ reads, after using its definition in Equation (2) and performing some simplifications

$$\begin{aligned}E\{W_T|\mathbf{S}\}= {} & W_{T-1}\cdot(1+r)+h(t_{h,T-1})\\ &+\frac{1}{\alpha}\cdot\left(E\{\mathbf{P}_T\}-(1+r)\cdot\mathbf{P}_{T-1}\right)'\mathbf{C}^{-1}_{P_T|S}(\mathbf{t}_{T-1})\left(E\{\mathbf{P}_T\}-(1+r)\cdot\mathbf{P}_{T-1}\right)\\ &+2\cdot\frac{1}{\alpha}\cdot\left(E\{\mathbf{P}_T\}-(1+r)\cdot\mathbf{P}_{T-1}\right)'\mathbf{C}^{-1}_{P_T|S}(\mathbf{t}_{T-1})\,\mathbf{COV}_{P_TS}(\mathbf{t}_{T-1})\mathbf{C}_S^{-1}(\mathbf{S}-E\{\mathbf{S}\})\\ &+\frac{1}{\alpha}\cdot(\mathbf{S}-E\{\mathbf{S}\})'\mathbf{C}_S^{-1}\,\mathbf{COV}'_{P_TS}(\mathbf{t}_{T-1})\mathbf{C}^{-1}_{P_T|S}(\mathbf{t}_{T-1})\,\mathbf{COV}_{P_TS}(\mathbf{t}_{T-1})\mathbf{C}_S^{-1}(\mathbf{S}-E\{\mathbf{S}\})\end{aligned} \tag{A1.3}$$

$\mathrm{var}(W_T|\mathbf{S})$ reads, after using its definition in Equation (3) and performing some simplifications

$$\begin{aligned}\mathrm{var}(W_T|\mathbf{S})= {} & \frac{1}{\alpha^2}\cdot\left(E\{\mathbf{P}_T\}-(1+r)\cdot\mathbf{P}_{T-1}\right)'\mathbf{C}^{-1}_{P_T|S}(\mathbf{t}_{T-1})\left(E\{\mathbf{P}_T\}-(1+r)\cdot\mathbf{P}_{T-1}\right)\\ &+2\cdot\frac{1}{\alpha^2}\cdot\left(E\{\mathbf{P}_T\}-(1+r)\cdot\mathbf{P}_{T-1}\right)'\mathbf{C}^{-1}_{P_T|S}(\mathbf{t}_{T-1})\,\mathbf{COV}_{P_TS}(\mathbf{t}_{T-1})\mathbf{C}_S^{-1}(\mathbf{S}-E\{\mathbf{S}\})\\ &+\frac{1}{\alpha^2}\cdot(\mathbf{S}-E\{\mathbf{S}\})'\mathbf{C}_S^{-1}\,\mathbf{COV}'_{P_TS}(\mathbf{t}_{T-1})\mathbf{C}^{-1}_{P_T|S}(\mathbf{t}_{T-1})\,\mathbf{COV}_{P_TS}(\mathbf{t}_{T-1})\mathbf{C}_S^{-1}(\mathbf{S}-E\{\mathbf{S}\})\end{aligned} \tag{A1.4}$$

Inserting the expected value from Equation (A1.3) and the variance from Equation (A1.4) into Equation (A1.1), we obtain for the expected utility (A1.1)

$$E\left\{-\frac{1}{\alpha}\cdot e^{-\alpha\cdot W_T}\middle|\mathbf{S}\right\} = -\frac{1}{\alpha}\cdot e^{-\alpha\cdot W_{T-1}\cdot(1+r)-\alpha\cdot h\left(t_{h,T-1}\right)} \tag{A1.5}$$

$$\cdot\exp\left\{-\frac{1}{2}\cdot\left(E\{\mathbf{P}_T\}-(1+r)\cdot\mathbf{P}_{T-1}\right)'\mathbf{C}^{-1}_{P_T|S}(\mathbf{t}_{T-1})\left(E\{\mathbf{P}_T\}-(1+r)\cdot\mathbf{P}_{T-1}\right)\right.$$

$$-\left(E\{\mathbf{P}_T\}-(1+r)\cdot\mathbf{P}_{T-1}\right)'\mathbf{C}^{-1}_{P_T|S}(\mathbf{t}_{T-1})\mathbf{COV}_{P_TS}(\mathbf{t}_{T-1})\mathbf{C}^{-1}_S\left(\mathbf{S}-E\{\mathbf{S}\}\right)$$

$$\left.-\frac{1}{2}\cdot\left(\mathbf{S}-E\{\mathbf{S}\}\right)'\mathbf{C}^{-1}_S\mathbf{COV}'_{P_TS}(\mathbf{t}_{T-1})\mathbf{C}^{-1}_{P_T|S}(\mathbf{t}_{T-1})\mathbf{COV}_{P_TS}(\mathbf{t}_{T-1})\mathbf{C}^{-1}_S\left(\mathbf{S}-E\{\mathbf{S}\}\right)\right\}$$

A.1.2. Second step: Calculation of the expected value of the outer problem (6) using the expression for the inner problem (A1.5)

To calculate the expected value of the outer problem and, thus, to have a foundation for determining the optimal time allocation, the following expectation with respect to signals **S** must be computed:

$$E\left\{E\left\{-\frac{1}{\alpha}\cdot e^{-\alpha\cdot W_T}\middle|\mathbf{S}\right\}\right\} \tag{A1.6}$$

Calculations will be simplified by switching from normally distributed variables **S** to standard normally distributed variables $\mathbf{Y}_S$ by using the transformation $\mathbf{S}=E\{\mathbf{S}\}+\left[\left(\text{cholesky}\left(\mathbf{C}_S^{-1}\right)\right)'\right]^{-1}\mathbf{Y}_S$.[11] For that reason, it holds

$$E\left\{E\left\{-\frac{1}{\alpha}\cdot e^{-\alpha\cdot W_T}\middle|\mathbf{S}\right\}\right\} = -\frac{1}{\alpha}\cdot e^{-\alpha\cdot W_{T-1}\cdot(1+r)-\alpha\cdot h\left(t_{h,T-1}\right)} \tag{A1.7}$$

$$\cdot\exp\left\{-\frac{1}{2}\cdot\left(E\{\mathbf{P}_T\}-(1+r)\cdot\mathbf{P}_{T-1}\right)'\mathbf{C}^{-1}_{P_T|S}(\mathbf{t}_{T-1})\left(E\{\mathbf{P}_T\}-(1+r)\cdot\mathbf{P}_{T-1}\right)\right\}$$

[11] This transformation stems from the consideration that $\left(\mathbf{S}-E\{\mathbf{S}\}\right)'\text{cholesky}\left(\mathbf{C}_S^{-1}\right)\left(\text{cholesky}\left(\mathbf{C}_S^{-1}\right)\right)'\left(\mathbf{S}-E\{\mathbf{S}\}\right)$ should be transformed into $\mathbf{Y}_S'\,\text{Id}\,\mathbf{Y}_S$ and has in the event of just one random variable the well-known manifestation $S=E\{S\}+\text{std}(S)\cdot Y_S$.

$$\cdot \mathrm{E}\Big\{-\left(\mathrm{E}\{\mathbf{P}_T\}-(1+r)\cdot\mathbf{P}_{T-1}\right)'\mathbf{C}^{-1}_{P_T|S}(\mathbf{t}_{T-1})\,\mathrm{cholesky}\left(\mathbf{C}_S^{-1}\right)\mathbf{Y}_S$$

$$-\frac{1}{2}\cdot\mathbf{Y}'_S\,\mathrm{cholesky}\left(\mathbf{C}_S^{-1}\right)\mathbf{COV}'_{P_TS}(\mathbf{t}_{T-1})\,\mathbf{C}^{-1}_{P_T|S}(\mathbf{t}_{T-1})\,\mathbf{COV}_{P_TS}(\mathbf{t}_{T-1})\,\mathrm{cholesky}\left(\mathbf{C}_S^{-1}\right)\mathbf{Y}_S\Big\}$$

Using the signals' m-dimensional standard normal density function to calculate the expected value in Equation (A1.7) (last two lines of Equation (A1.7)), Equation (A1.7) can be written as

$$\mathrm{E}\left\{\mathrm{E}\left\{-\frac{1}{\alpha}\cdot e^{-\alpha\cdot W_T}\middle|\mathbf{S}\right\}\right\} = -\frac{1}{\alpha}\cdot e^{-\alpha\cdot W_{T-1}\cdot(1+r)-\alpha\cdot h\left(t_{h,T-1}\right)} \tag{A1.8}$$

$$\cdot\exp\left\{-\frac{1}{2}\cdot\left(\mathrm{E}\{\mathbf{P}_T\}-(1+r)\cdot\mathbf{P}_{T-1}\right)'\mathbf{C}^{-1}_{P_T|S}(\mathbf{t}_{T-1})\left(\mathrm{E}\{\mathbf{P}_T\}-(1+r)\cdot\mathbf{P}_{T-1}\right)\right\}$$

$$\cdot\int_{-\infty}^{\infty}\cdots\int_{-\infty}^{\infty}\frac{1}{\sqrt[m]{2\cdot\pi}}\cdot\exp\left(-\frac{1}{2}\mathbf{y}'_S\,\mathrm{Id}\,\mathbf{y}_S\right)$$

$$\cdot\exp\left(-\left(\mathrm{E}\{\mathbf{P}_T\}-(1+r)\cdot\mathbf{P}_{T-1}\right)'\mathbf{C}^{-1}_{P_T|S}(\mathbf{t}_{T-1})\,\mathbf{COV}_{P_TS}(\mathbf{t}_{T-1})\,\mathrm{cholesky}\left(\mathbf{C}_S^{-1}\right)\mathbf{y}_S\right)$$

$$\cdot\exp\left(-\frac{1}{2}\cdot\mathbf{y}'_S\left(\mathrm{cholesky}\left(\mathbf{C}_S^{-1}\right)\right)'\mathbf{COV}'_{P_TS}(\mathbf{t}_{T-1})\mathbf{C}^{-1}_{P_T|S}(\mathbf{t}_{T-1})\mathbf{COV}_{P_TS}(\mathbf{t}_{T-1})\mathrm{cholesky}\left(\mathbf{C}_S^{-1}\right)\mathbf{y}_S\right)$$

$$\cdot d\,y_{S_1}\cdots d\,y_{S_m}$$

where $\mathbf{y}_S$ denotes the realizations of the random variables $\mathbf{Y}_S$, and Id is the identity matrix.

To compute the integrals in Equation (A1.8), we strive to transform the integrants in Equation (A1.8) to density functions so that the integrals equal 1. This requires, first, that the terms $\exp\left(-\frac{1}{2}\mathbf{y}'_S\,\mathrm{Id}\,\mathbf{y}_S\right)$ and $\exp\left(-\frac{1}{2}\cdot\mathbf{y}'_S\left(\mathrm{cholesky}\left(\mathbf{C}_S^{-1}\right)\right)'\mathbf{COV}'_{P_TS}(\mathbf{t}_{T-1})\mathbf{C}^{-1}_{P_T|S}(\mathbf{t}_{T-1})\mathbf{COV}_{P_TS}(\mathbf{t}_{T-1})\right.$ $\left.\mathrm{cholesky}\left(\mathbf{C}_S^{-1}\right)\mathbf{y}_S\right)$ are combined and, second, that they are transformed into a new standard normally distributed random variable with realizations $\mathbf{z}_S$; formally

$$\exp\left(-\frac{1}{2}\mathbf{z}'_S\,\mathrm{Id}\,\mathbf{z}_S\right)\equiv\exp\left(-\frac{1}{2}\mathbf{y}'_S\,\mathrm{Id}\,\mathbf{y}_S\right) \tag{A1.9}$$

$$\cdot \exp\left(-\frac{1}{2}\cdot \mathbf{y}_S' \left(\text{cholesky}\left(\mathbf{C}_S^{-1}\right)\right)\mathbf{COV}_{P_T S}'(\mathbf{t}_{T-1})\mathbf{C}_{P_T|S}^{-1}(\mathbf{t}_{T-1})\mathbf{COV}_{P_T S}(\mathbf{t}_{T-1})\text{cholesky}\left(\mathbf{C}_S^{-1}\right)\mathbf{y}_S\right)$$

$$= \exp\left(-\frac{1}{2}\mathbf{y}_S' \underbrace{\left[\text{Id} + \left(\text{cholesky}\left(\mathbf{C}_S^{-1}\right)\right)' \mathbf{COV}_{P_T S}'(\mathbf{t}_{T-1})\mathbf{C}_{P_T|S}^{-1}(\mathbf{t}_{T-1})\mathbf{COV}_{P_T S}(\mathbf{t}_{T-1})\text{cholesky}\left(\mathbf{C}_S^{-1}\right)\right]}_{\equiv \mathbf{B}(\mathbf{t}_{T-1})}\mathbf{y}_S\right)$$

Applying the argument set out in footnote 11 leads to $\mathbf{y}_S = \left[\left(\text{cholesky}\left(\mathbf{B}(\mathbf{t}_{T-1})\right)\right)'\right]^{-1}\mathbf{z}_S$. Along with changing the exponent from $\mathbf{y}_S$ to $\mathbf{z}_S$, we also need to adapt the integration variables. Note that $d\,y_{S_1}$ contains the first row of the vector $\mathbf{y}_S$ and thus equals the first row of $\mathbf{y}_S = \left[\left(\text{cholesky}\left(\mathbf{B}(\mathbf{t}_{T-1})\right)\right)'\right]^{-1}\mathbf{z}_S$. For that reason, $d\,y_{S_1}\cdots d\,y_{S_m}$ can be obtained by multiplying the elements of the main diagonal of $\left[\left(\text{cholesky}\left(\mathbf{B}(\mathbf{t}_{T-1})\right)\right)'\right]^{-1}$ and switching to $d\,z_{S_1}\cdots d\,z_{S_m}$.

Therefore, the next problem is writing “multiplying the elements of the main diagonal” of $\left[\left(\text{cholesky}\left(\mathbf{B}(\mathbf{t}_{T-1})\right)\right)'\right]^{-1}$ in a more concise form. From the transformation of normally distributed random variables into standard normally distributed random variables (see the argument in footnote 11 and the text accompanying same), we know that the product of the elements of the main diagonal of $\left[\left(\text{cholesky}\left(\mathbf{C}_S^{-1}\right)\right)'\right]^{-1}$ equals $\sqrt{\det(\mathbf{C}_S)}$, where det(.) denotes the determinant of a matrix. By analogy, we obtain for the product of the elements of the main diagonal of $\left[\left(\text{cholesky}\left(\mathbf{B}(\mathbf{t}_{T-1})\right)\right)'\right]^{-1}$ the shorter expression $\sqrt{\det\left(\mathbf{B}^{-1}(\mathbf{t}_{T-1})\right)}$.

Based on the above findings, Equation (A1.8) simplifies to

$$E\left\{E\left\{-\frac{1}{\alpha}\cdot e^{-\alpha\cdot W_T}\middle|\mathbf{S}\right\}\right\} = -\frac{1}{\alpha}\cdot e^{-\alpha\cdot W_{T-1}\cdot(1+r)-\alpha\cdot h\left(t_{h,T-1}\right)} \tag{A1.10}$$

$$\cdot \exp\left\{-\frac{1}{2}\cdot\left(E\{\mathbf{P}_T\}-(1+r)\cdot\mathbf{P}_{T-1}\right)'\mathbf{C}_{P_T|S}^{-1}(\mathbf{t}_{T-1})\left(E\{\mathbf{P}_T\}-(1+r)\cdot\mathbf{P}_{T-1}\right)\right\}$$

$$\cdot \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} \frac{1}{\sqrt[m]{2 \cdot \pi}} \cdot \exp\left(-\frac{1}{2} \mathbf{z}_S' \, \mathrm{Id} \, \mathbf{z}_S \right)$$

$$\cdot \exp\left(-\left(\mathrm{E}\{\mathbf{P}_T\} - (1+r) \cdot \mathbf{P}_{T-1}\right)' \mathbf{C}^{-1}_{P_T|S}(\mathbf{t}_{T-1}) \, \mathbf{COV}_{P_T S}(\mathbf{t}_{T-1}) \, \mathrm{cholesky}\left(\mathbf{C}_S^{-1}\right) \right.$$

$$\left. \left[\left(\mathrm{cholesky}(\mathbf{B}(\mathbf{t}_{T-1}))\right)' \right]^{-1} \mathbf{z}_S \right)$$

$$\cdot \, d z_{S_1} \cdots d z_{S_m} \cdot \sqrt{\det\left(\mathbf{B}^{-1}(\mathbf{t}_{T-1})\right)}$$

Finally, to finish the transformation to density functions, we need to complete the square of the exponent in Equation (A1.10). Define $\mathbf{K}'(\mathbf{t}_{T-1}) \equiv \mathrm{E}\{\mathbf{P}_T - (1+r) \cdot \mathbf{P}_{T-1}\}' \mathbf{C}^{-1}_{P_T|S}(\mathbf{t}_{T-1}) \, \mathbf{COV}_{P_T S}$ $\mathrm{cholesky}\left(\mathbf{C}_S^{-1}\right)\left[\left(\mathrm{cholesky}(\mathbf{B}(\mathbf{t}_{T-1}))\right)'\right]^{-1}$, then the square can be completed by adding $\pm \frac{1}{2} \mathbf{K}'(\mathbf{t}_{T-1}) \mathbf{K}(\mathbf{t}_{T-1})$ to the exponent of the term in the second and third to last lines of Equation (A1.10). After these transformations, Equation (A1.10) reads

$$\mathrm{E}\left\{ \mathrm{E}\left\{ -\frac{1}{\alpha} \cdot e^{-\alpha \cdot W_T} \middle| \mathbf{S} \right\} \right\} = -\frac{1}{\alpha} \cdot e^{-\alpha \cdot W_{T-1} \cdot (1+r) - \alpha \cdot h(t_{h,T-1})} \tag{A1.11}$$

$$\cdot \exp\left\{ -\frac{1}{2} \cdot \left(\mathrm{E}\{\mathbf{P}_T\} - (1+r) \cdot \mathbf{P}_{T-1}\right)' \mathbf{C}^{-1}_{P_T|S}(\mathbf{t}_{T-1}) \left(\mathrm{E}\{\mathbf{P}_T\} - (1+r) \cdot \mathbf{P}_{T-1}\right) \right\}$$

$$\cdot \, e^{\frac{1}{2} \mathbf{K}'(\mathbf{t}_{T-1}) \mathbf{K}(\mathbf{t}_{T-1})}$$

$$\cdot \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} \frac{1}{\sqrt[m]{2 \cdot \pi}} \cdot \exp\left(-\frac{1}{2} \left(\mathbf{z}_S + \mathbf{K}(\mathbf{t}_{T-1})\right)' \, \mathrm{Id} \, \left(\mathbf{z}_S + \mathbf{K}(\mathbf{t}_{T-1})\right) \right)$$

$$\cdot \, d z_{S_1} \cdots d z_{S_m} \cdot \sqrt{\det\left(\mathbf{B}^{-1}(\mathbf{t}_{T-1})\right)}$$

which yields

$$\mathrm{E}\left\{ \mathrm{E}\left\{ -\frac{1}{\alpha} \cdot e^{-\alpha \cdot W_T} \middle| \mathbf{S} \right\} \right\} = -\frac{1}{\alpha} \cdot e^{-\alpha \cdot W_{T-1} \cdot (1+r) - \alpha \cdot h(t_{h,T-1})} \tag{A1.12}$$

$$\cdot \exp\left\{-\frac{1}{2}\cdot\left(\mathrm{E}\{\mathbf{P}_{\mathrm{T}}\}-(1+\mathrm{r})\cdot\mathbf{P}_{\mathrm{T}-1}\right)'\mathbf{C}^{-1}_{\mathrm{P_T|S}}(\mathbf{t}_{\mathrm{T}-1})\left(\mathrm{E}\{\mathbf{P}_{\mathrm{T}}\}-(1+\mathrm{r})\cdot\mathbf{P}_{\mathrm{T}-1}\right)\right\}$$

$$\cdot e^{\frac{1}{2}\mathbf{K}'(\mathbf{t}_{\mathrm{T}-1})\mathbf{K}(\mathbf{t}_{\mathrm{T}-1})}\cdot\sqrt{\det\left(\mathbf{B}^{-1}(\mathbf{t}_{\mathrm{T}-1})\right)}$$

A.1.3. Third step: Simplification of the components of Equation (A1.12) to derive Equation (10)

Problem (10) distinguishes itself from Equation (A1.12) in two respects. On the one hand, the second line of Equation (A1.12) must be merged with $\frac{1}{2}\mathbf{K}'(\mathbf{t}_{\mathrm{T}-1})\mathbf{K}(\mathbf{t}_{\mathrm{T}-1})$; on the other hand, $\mathbf{B}(\mathbf{t}_{\mathrm{T-1}})$ must be simplified.

Tedious calculations[12] lead to

$$\exp\left\{-\frac{1}{2}\cdot\left(\mathrm{E}\{\mathbf{P}_{\mathrm{T}}\}-(1+\mathrm{r})\cdot\mathbf{P}_{\mathrm{T}-1}\right)'\mathbf{C}^{-1}_{\mathrm{P_T|S}}(\mathbf{t}_{\mathrm{T}-1})\left(\mathrm{E}\{\mathbf{P}_{\mathrm{T}}\}-(1+\mathrm{r})\cdot\mathbf{P}_{\mathrm{T}-1}\right)\right\} \tag{A1.13}$$

$$\cdot e^{\frac{1}{2}\mathbf{K}'(\mathbf{t}_{\mathrm{T}-1})\mathbf{K}(\mathbf{t}_{\mathrm{T}-1})} = e^{-\frac{1}{2}\cdot\mathrm{E}\{\mathbf{P}_{\mathrm{T}}-(1+\mathrm{r})\cdot\mathbf{P}_{\mathrm{T}-1}\}'\mathbf{C}^{-1}_{\mathrm{P_T}}\mathrm{E}\{\mathbf{P}_{\mathrm{T}}-(1+\mathrm{r})\cdot\mathbf{P}_{\mathrm{T}-1}\}}$$

To express $\mathbf{B}(\mathbf{t}_{\mathrm{T}-1})\equiv \mathrm{Id}+\left(\mathrm{cholesky}\left(\mathbf{C}_{\mathrm{S}}^{-1}\right)\right)'\mathbf{COV}'_{\mathrm{P_TS}}(\mathbf{t}_{\mathrm{T}-1})\,\mathbf{C}^{-1}_{\mathrm{P_T|S}}(\mathbf{t}_{\mathrm{T}-1})\,\mathbf{COV}_{\mathrm{P_TS}}(\mathbf{t}_{\mathrm{T}-1})\,\mathrm{cholesky}\left(\mathbf{C}_{\mathrm{S}}^{-1}\right)$ with the help of correlation coefficients, proceed as follows. Write all variance/covariance or covariance matrices as the product of the matrix of standard deviations and the matrix of correlation coefficients, i.e., $\mathbf{C}_{\mathrm{S}}=\mathrm{diag}_{\mathrm{S}}\rho_{\mathrm{SS}}$, $\mathbf{COV}_{\mathrm{P_TS}}(\mathbf{t}_{\mathrm{T}-1})=\mathrm{diag}_{\mathrm{P_T}}\boldsymbol{\rho}_{\mathrm{P_TS}}(\mathbf{t}_{\mathrm{T}-1})\,\mathrm{diag}_{\mathrm{S}}$, and $\mathbf{C}_{\mathrm{P_T}}=\mathrm{diag}_{\mathrm{P_T}}\boldsymbol{\rho}_{\mathrm{P_TP_T}}$, where $\mathrm{diag}_{\mathrm{PT}}$ denotes the diagonal matrix of standard deviations of stock prices, $\mathrm{diag}_{\mathrm{S}}$ the diagonal matrix of standard deviations of signals, and $\boldsymbol{\rho}$ denotes the matrix of correlation coefficients between random variables.

With these substitutions, we obtain

$$\mathbf{B}(\mathbf{t}_{\mathrm{T}-1})\equiv \quad \mathrm{Id} \tag{A1.14}$$

$$+\left(\mathrm{cholesky}\left(\mathbf{C}_{\mathrm{S}}^{-1}\right)\right)'\mathrm{diag}_{\mathrm{S}}\,\boldsymbol{\rho}'_{\mathrm{P_TS}}(\mathbf{t}_{\mathrm{T}-1})\,\mathrm{diag}_{\mathrm{P_T}}\mathrm{C}^{-1}_{\mathrm{P_T|S}}(\mathbf{t}_{\mathrm{T}-1})\,\mathrm{diag}_{\mathrm{P_T}}\,\boldsymbol{\rho}_{\mathrm{P_TS}}(\mathbf{t}_{\mathrm{T}-1})\,\mathrm{diag}_{\mathrm{S}}\ \mathrm{cholesky}\left(\mathbf{C}_{\mathrm{S}}^{-1}\right)$$

with

$$\mathbf{C}_{P_T|S} = \mathrm{diag}_{P_T}\boldsymbol{\rho}_{P_TP_T}\mathrm{diag}_{P_T} - \mathrm{diag}_{P_T}\boldsymbol{\rho}_{P_TS}(\mathbf{t}_{T-1})\boldsymbol{\rho}_{SS}^{-1}\boldsymbol{\rho}_{P_TS}(\mathbf{t}_{T-1})\,\mathrm{diag}_{P_T}$$

and

$$\mathrm{cholesky}\left(\mathbf{C}_S^{-1}\right) = \mathrm{cholesky}\left(\mathrm{diag}_S^{-1}\,\boldsymbol{\rho}_{SS}^{-1}\,\mathrm{diag}_S^{-1}\right)$$

Since[13] $\mathrm{cholesky}\left(\mathbf{C}_S^{-1}\right) = \mathrm{diag}_S^{-1}\mathrm{cholesky}\left(\boldsymbol{\rho}_{SS}^{-1}\right)$, Equation (A1.14) can be written as

$$\mathbf{B}(\mathbf{t}_{T-1}) \equiv \mathrm{Id} + \left(\mathrm{cholesky}\left(\boldsymbol{\rho}_{SS}^{-1}\right)\right)' \boldsymbol{\rho}'_{P_TS}(\mathbf{t}_{T-1})\,\mathrm{diag}_{P_T}\mathrm{C}_{P_T|S}^{-1}(\mathbf{t}_{T-1})\,\mathrm{diag}_{P_T}\,\boldsymbol{\rho}_{P_TS}(\mathbf{t}_{T-1})\,\mathrm{cholesky}\left(\boldsymbol{\rho}_{SS}^{-1}\right) \quad \text{(A1.15)}$$

with

$$\mathbf{C}_{P_T|S}(\mathbf{t}_{T-1}) = \mathrm{diag}_{P_T}\left[\boldsymbol{\rho}_{P_TP_T} - \boldsymbol{\rho}_{P_TS}(\mathbf{t}_{T-1})\boldsymbol{\rho}_{SS}^{-1}\boldsymbol{\rho}_{P_TS}(\mathbf{t}_{T-1})\right]\mathrm{diag}_{P_T}$$

From Equation (A1.15) follows immediately the expression of $\mathbf{B}(\mathbf{t}_{T-1})$ used in Equation (10).

A.1.4. Derivation of the special case of Equation (11): Uncorrelated signals and uncorrelated stocks

In the event of uncorrelated signals, the correlation matrix of signals $\boldsymbol{\rho}_{SS}$ equals an identity matrix; the same is true for the correlation matrix of stocks $\boldsymbol{\rho}_{P_TP_T}$. Finally, $\boldsymbol{\rho}_{P_TS}(\mathbf{t}_{T-1})$ transforms into the diagonal matrix $\boldsymbol{\rho}_{P_TS}(\mathbf{t}_{T-1}) = \begin{pmatrix} \rho_{P_{1,T}S_1}(t_{1,S_1,T-1}) & 0 & \cdots \\ 0 & \rho_{P_{2,T}S_2}(t_{2,S_2,T-1}) & 0 \\ \vdots & 0 & \ddots \end{pmatrix}$. Based on these simplifications, $\mathbf{C}_{P_T|S}$ reads $\mathbf{C}_{P_T|S} = \mathrm{diag}_{P_T}\begin{pmatrix} 1-\rho^2_{P_{1,T}S_1}(t_{1,S_1,T-1}) & 0 & \cdots \\ 0 & 1-\rho^2_{P_{2,T}S_2}(t_{2,S_2,T-1}) & 0 \\ \vdots & 0 & \ddots \end{pmatrix}\mathrm{diag}_{P_T}$.

Inserting this expression for $\mathbf{C}_{P_T|S}$ into Equation (A1.15), yields

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12 These calculations are too lengthy to be set out here; however, they are available from the authors upon request as Maple files.

13 This can be seen as follows: $\mathrm{cholesky}\left(\mathbf{C}_S^{-1}\right)\left(\mathrm{cholesky}\left(\mathbf{C}_S^{-1}\right)\right)' = \mathbf{C}_S^{-1}$ by definition. Using the pretended relation $\mathrm{diag}_S^{-1}\mathrm{cholesky}\left(\boldsymbol{\rho}_{SS}^{-1}\right)$, we calculate $\mathrm{diag}_S^{-1}\mathrm{cholesky}\left(\boldsymbol{\rho}_{SS}^{-1}\right)\left(\mathrm{diag}_S^{-1}\mathrm{cholesky}\left(\boldsymbol{\rho}_{SS}^{-1}\right)\right)'$, which equals $\mathrm{diag}_S^{-1}\mathrm{cholesky}\left(\boldsymbol{\rho}_{SS}^{-1}\right)\left(\mathrm{cholesky}\left(\boldsymbol{\rho}_{SS}^{-1}\right)\right)'\mathrm{diag}_S^{-1}$, i.e., $\mathrm{diag}_S^{-1}\boldsymbol{\rho}_{SS}^{-1}\,\mathrm{diag}_S^{-1} = \mathbf{C}_S^{-1}$.

$$\mathbf{B}(\mathbf{t}_{T-1}) \equiv \mathrm{Id} + \begin{pmatrix} \rho_{P_{1,T}S_1}(t_{1,S_1,T-1}) & 0 & \cdots \\ 0 & \rho_{P_{2,T}S_2}(t_{2,S_2,T-1}) & 0 \\ \vdots & 0 & \ddots \end{pmatrix} \cdot \tag{A1.16}$$

$$\begin{pmatrix} 1-\rho^2_{P_{1,T}S_1}(t_{1,S_1,T-1}) & 0 & \cdots \\ 0 & 1-\rho^2_{P_{2,T}S_2}(t_{2,S_2,t-1}) & 0 \\ \vdots & 0 & \ddots \end{pmatrix}^{-1} \begin{pmatrix} \rho_{P_{1,T}S_1}(t_{1,S_1,T-1}) & 0 & \cdots \\ 0 & \rho_{P_{2,T}S_2}(t_{2,S_2,T-1}) & 0 \\ \vdots & 0 & \ddots \end{pmatrix}$$

In other words,

$$\mathbf{B}(\mathbf{t}_{T-1}) = \begin{pmatrix} 1 & 0 & \cdots \\ 0 & 1 & 0 \\ \vdots & 0 & \ddots \end{pmatrix} + \begin{pmatrix} \frac{\rho^2_{P_{1,T}S_1}(t_{1,S_1,T-1})}{1-\rho^2_{P_{1,T}S_1}(t_{1,S_1,T-1})} & 0 & \cdots \\ 0 & \frac{\rho^2_{P_{2,T}S_2}(t_{2,S_{12},T-1})}{1-\rho^2_{P_{2,T}S_2}(t_{2,S_{21},T-1})} & 0 \\ \vdots & 0 & \ddots \end{pmatrix} \tag{A1.17}$$

and finally

$$\mathbf{B}(\mathbf{t}_{T-1}) = \begin{pmatrix} \frac{1}{1-\rho^2_{P_{1,T}S_1}(t_{1,S_1,T-1})} & 0 & \cdots \\ 0 & \frac{1}{1-\rho^2_{P_{2,T}S_2}(t_{2,S_2,T-1})} & 0 \\ \vdots & 0 & \ddots \end{pmatrix} \tag{A1.18}$$

Calculating $\det\left(\mathbf{B}(\mathbf{t}_{T-1})^{-1}\right)$ with the help of Equation (A1.18), results in

$$\det\left(\mathbf{B}(\mathbf{t}_{T-1})^{-1}\right) = \left(1-\rho^2_{P_{1,T,}S_1}(t_{1,S_1,T-1})\right)\cdot\left(1-\rho^2_{P_{2,T}S_2}(t_{2,S_2,T-1})\right)\cdot\ldots \tag{A1.19}$$

Assuming a functional relationship between time investment and the correlation coefficient between stock prices and signals as $\rho_{P_{i,T}S_i}(t_{i,S_i,T-1}) = \sqrt{\frac{t_{i,S_i,T-1}}{X_{i,S_i,T-1}}}$, Equation (A1.19) simplifies to

$$\det\left(\mathbf{B}(\mathbf{t}_{T-1})^{-1}\right) = \left(1-\frac{t_{1,S_1,T-1}}{X_{1,S_1,T-1}}\right)\cdot\left(1-\frac{t_{2,S_2,T-1}}{X_{2,S_2,T-1}}\right)\cdot\ldots \tag{A1.20}$$

Furthermore, assume a bonus payment function that is linear in $t_{h,T-1}$:

$$h\left(t_{h,T-1}\right) = \frac{W_{max}}{\overline{T}_{h,T-1}}\cdot t_{h,T-1} \tag{A1.21}$$

Using the time constraint $t_{h,T-1} = \overline{T}_{T-1} - \sum_{i=1}^{n} t_{i,S_i,T-1}$ as well as Equations (A1.20) and (A1.21), the necessary condition for the time invested in the analysis of stock i (Equation (11)) simplifies in this special case to

$$\frac{\partial}{\partial t_{i,S_i,T-1}} = 0 = \alpha \frac{W_{max}}{\overline{T}_{h,T-1}} + \frac{1}{2} \cdot \frac{\left(1 - \frac{t_{1,S_1,T-1}}{X_{1,S_1,T-1}}\right) \cdot \left(1 - \frac{t_{2,S_2,T-1}}{X_{2,S_2,T-1}}\right) \cdot \ldots}{\left(1 - \frac{t_{1,S_1,T-1}}{X_{1,S_1,T-1}}\right) \cdot \left(1 - \frac{t_{2,S_2,T-1}}{X_{2,S_2,T-1}}\right) \cdot \left(1 - \frac{t_{i,S_i,T-1}}{X_{i,S_i,t-1}}\right) \cdot \ldots} \cdot \left(-\frac{1}{X_{i,S_i,T-1}}\right) \tag{A1.22}$$

that is,

$$1 - \frac{t_{i,S_i}}{X_{i,S_i}} = \frac{1}{2} \cdot \frac{\frac{1}{X_{i,S_i}}}{\alpha \frac{W_{max}}{\overline{T}_{h,T-1}}} \tag{A1.23}$$

and finally

$$t_{i,S_i,T-1} = X_{i,S_i,T-1} - \frac{1}{2} \cdot \frac{\overline{T}_{h,T-1}}{\alpha \cdot W_{max}} \tag{A1.24}$$

A.2. Optimal portfolio holdings and time allocations in the dynamic model of the numerical example

A.2.1. First step: Calculation of the inner problem at calendar time T – 1 of decision problem (5)

The inner problem at calendar time T – 1 of our two-period decision problem reads

$$\underset{\mathbf{N}_{T-1}}{\text{Max}}\ E\left\{ -\frac{1}{\alpha} \cdot e^{-\alpha \cdot W_T} \middle| \mathbf{S}_{T-1}, \mathbf{S}_{T-2}, W_{T-1} \right\} \tag{A2.1}$$

Key to its solution is calculating the distribution of W_T conditional on $\mathbf{S}_{T-1}$, $\mathbf{S}_{T-2}$, and W_{T-1}.

Using the n dimensional multinormal density of stock prices at calendar time T (see, e.g., Mardia/Kent/Bibby , 1992, p. 37), the expected value of Equation (A2.1) can be written as

$$-\frac{1}{\alpha}\cdot\int_{-\infty}^{\infty}\cdots\int_{-\infty}^{\infty}\frac{1}{\sqrt{\det\left(2\cdot\pi\cdot\mathbf{C}_{\mathbf{P}_T|\mathbf{S}_{T-1},\mathbf{S}_{T-2},W_{T-1}}\right)}} \tag{A2.2}$$

$$\cdot\exp\left(-\frac{1}{2}\cdot\left(\mathbf{p}_T-\mathbf{E}_{\mathbf{P}_T|\mathbf{S}_{T-1},\mathbf{S}_{T-2},W_{T-1}}\right)'\mathbf{C}^{-1}_{\mathbf{P}_T|\mathbf{S}_{T-1},\mathbf{S}_{T-2},W_{T-1}}\left(\mathbf{p}_T-\mathbf{E}_{\mathbf{P}_T|\mathbf{S}_{T-1},\mathbf{S}_{T-2},W_{T-1}}\right)\right)$$

$$\cdot e^{-\alpha\cdot W_T}d\,p_{1,T}\cdots d\,p_{n,T}$$

where $d\,p_{i,T}$ denotes the realization of the random variable price of stock i at calendar time T and $\mathbf{p}_T$ is the vector of the realization of stock prices at calendar time T

with

$$\mathbf{E}_{\mathbf{P}_T|\mathbf{S}_{T-1},\mathbf{S}_{T-2},W_{T-1}}=E\{\mathbf{P}_T\}+\mathbf{COV}_{P_T S}(\mathbf{t}_{T-2},\mathbf{t}_{T-1})\mathbf{C}_S^{-1}(\mathbf{S}-E\{\mathbf{S}\})$$

$$\mathbf{C}_{\mathbf{P}_T|\mathbf{S}_{T-1},\mathbf{S}_{T-2},W_{T-1}}=\mathbf{C}_{P_T}-\mathbf{COV}_{P_T S}(\mathbf{t}_{T-2},\mathbf{t}_{T-1})\mathbf{C}_S^{-1}\mathbf{COV}'_{P_T S}(\mathbf{t}_{T-2},\mathbf{t}_{T-1})$$

and

$$\mathbf{S}'=\begin{pmatrix}S_{1,T-1} & S_{2,T-2} & \cdots & S_{1,T-2} & S_{2,t-2} & \cdots & W_{T-1}\end{pmatrix}$$

Assuming two signals, the abstract conditional expected values, variances, and covariances can be particularized as follows:

$$\mathbf{COV}'_{P_T S}(\mathbf{t}_{T-2},\mathbf{t}_{T-1})=\begin{pmatrix}\operatorname{cov}(P_{1,T},S_{1,T-1})(\mathbf{t}_{T-2},\mathbf{t}_{T-1}) & \operatorname{cov}(P_{2,T},S_{1,T-1})(\mathbf{t}_{T-2},\mathbf{t}_{T-1})\\ \operatorname{cov}(P_{1,T},S_{2,T-1})(\mathbf{t}_{T-2},\mathbf{t}_{T-1}) & \operatorname{cov}(P_{2,T},S_{2,T-1})(\mathbf{t}_{T-2},\mathbf{t}_{T-1})\\ \operatorname{cov}(P_{1,T},S_{1,T-2})(\mathbf{t}_{T-2},\mathbf{t}_{T-1}) & \operatorname{cov}(P_{2,T},S_{1,T-2})(\mathbf{t}_{T-2},\mathbf{t}_{T-1})\\ \operatorname{cov}(P_{1,T},S_{2,T-2})(\mathbf{t}_{T-2},\mathbf{t}_{T-1}) & \operatorname{cov}(P_{2,T},S_{2,T-2})(\mathbf{t}_{T-2},\mathbf{t}_{T-1})\\ \operatorname{cov}(P_{1,T},W_{T-1})(\mathbf{t}_{T-2},\mathbf{t}_{T-1}) & \operatorname{cov}(P_{2,T},W_{T-1})(\mathbf{t}_{T-2},\mathbf{t}_{T-1})\end{pmatrix}$$

$$\mathbf{C}_S=$$

$$\begin{pmatrix}\operatorname{var}(S_{1,T-1}) & \operatorname{cov}(S_{1,T-1},S_{2,T-1}) & \operatorname{cov}(S_{1,T-1},S_{1,T-2}) & \operatorname{cov}(S_{1,T-1},S_{2,T-2}) & \operatorname{cov}(S_{1,T-1},W_{T-1})\\ \operatorname{cov}(S_{2,T-1},S_{1,T-1}) & \operatorname{var}(S_{2,T-1}) & \operatorname{cov}(S_{2,T-1},S_{1,T-2}) & \operatorname{cov}(S_{2,T-1},S_{2,T-2}) & \operatorname{cov}(S_{2,T-1},W_{T-1})\\ \operatorname{cov}(S_{1,T-2},S_{1,T-1}) & \operatorname{cov}(S_{1,T-2},S_{2,T-1}) & \operatorname{var}(S_{1,T-2}) & \operatorname{cov}(S_{1,T-2},S_{2,T-2}) & \operatorname{cov}(S_{1,T-2},W_{T-1})\\ \operatorname{cov}(S_{2,T-2},S_{1,T-1}) & \operatorname{cov}(S_{2,T-2},S_{2,T-1}) & \operatorname{cov}(S_{2,T-2},S_{1,T-2}) & \operatorname{var}(S_{2,T-2}) & \operatorname{cov}(S_{2,T-2},W_{T-1})\\ \operatorname{cov}(W_{T-1},S_{1,T-1}) & \operatorname{cov}(W_{T-1},S_{2,T-1}) & \operatorname{cov}(W_{T-1},S_{1,T-2}) & \operatorname{cov}(W_{T-1},S_{2,T-2}) & \operatorname{var}(W_{T-1})\end{pmatrix}$$

where $\text{cov}(S_{1,T-1}, W_{T-1}) = \text{cov}(S_{2,T-1}, W_{T-1}) = 0$ because signals at calendar time T – 1 influence future wealth, but not wealth at the same calendar time when signal realizations become observable. $S_{j,T-i}$ denotes the random variable signal S_j at calendar time T – i, $P_{j,T-i}$ price of stock j at calendar time T – i, and ($\mathbf{t}_{T-i}$) portrays the dependence of covariances on the time invested in stock analysis at calendar time T – i.

Differentiation of Equation (A2.2) with respect to $\mathbf{N}_{T-1}$ yields the optimal portfolio numbers at calendar time T – 1: $\mathbf{N}^*_{T-1}(\mathbf{S}_{T-2}, \mathbf{S}_{T-1}, W_{T-1})$.

A.2.2. Second step: Calculation of the outer problem at calendar time T – 1 of decision problem (5)

The outer problem at calendar time T – 1 of our two-period decision problem reads

$$\underset{\mathbf{t}_{T-1}}{\text{Max}}\ E\left\{-\frac{1}{\alpha}\cdot e^{-\alpha\cdot W_T}\,\middle|\,\mathbf{S}_{T-2}, W_{T-1}\right\} \tag{A2.3}$$

where W_T contains $\mathbf{N}^*_{T-1}(\mathbf{S}_{T-2}, \mathbf{S}_{T-1}, W_{T-1})$.

Using the m dimensional multinormal density of signals at calendar time T – 1, the expected value of Equation (A2.3) can be written as

$$-\frac{1}{\alpha}\cdot\int_{-\infty}^{\infty}\cdots\int_{-\infty}^{\infty}\frac{1}{\sqrt{\det\left(2\cdot\pi\cdot\mathbf{C}_{\mathbf{P}_T|\mathbf{S}_{T-2},W_{T-1}}\right)}} \tag{A2.4}$$

$$\cdot\exp\left(-\frac{1}{2}\cdot\left(\mathbf{s}_{T-1}-\mathbf{E}_{\mathbf{S}_{T-1}|\mathbf{S}_{T-2},W_{T-1}}\right)'\mathbf{C}^{-1}_{\mathbf{S}_{T-1}|\mathbf{S}_{T-2},W_{T-1}}\left(\mathbf{s}_{T-1}-\mathbf{E}_{\mathbf{S}_{T-1}|\mathbf{S}_{T-2},W_{T-1}}\right)\right)$$

$$\cdot e^{-\alpha\cdot W_T}\,d\,s_{1,T-1}\cdots d\,s_{m,T-1}$$

where $d\,s_{i,T-1}$ denotes the realization of the random signal S_i at calendar time T – 1 and $\mathbf{s}_{T-1}$ is the vector of the realization of signals at calendar time T – 1,

with

$$\mathbf{E}_{\mathbf{S}_{T-1}|\mathbf{S}_{T-2},W_{T-1}} = E\{\mathbf{S}_{T-1}\} + \mathbf{COV}_{S_{T-1},S}\mathbf{C}_S^{-1}(\mathbf{S} - E\{\mathbf{S}\})$$

$$\mathbf{C}_{\mathbf{S}_{T-1}|\mathbf{S}_{T-2},W_{T-1}} = \mathbf{C}_{S_{T-1}} - \mathbf{COV}_{S_{T-1}S}\mathbf{C}_S^{-1}\mathbf{COV}'_{S_{T-1}S}$$

and

$$\mathbf{S}' = \begin{pmatrix} S_{1,T-2} & S_{2,T-2} & W_{T-1} \end{pmatrix}$$

$$\mathbf{COV}_{S_{T-1},S} = \begin{pmatrix} \operatorname{cov}(S_{1,T-1}, S_{1,T-2}) & \operatorname{cov}(S_{1,T-1}, S_{2,T-2}) & \operatorname{cov}(S_{1,T-1}, W_{T-1}) \\ \operatorname{cov}(S_{2,T-1}, S_{1,T-2}) & \operatorname{cov}(S_{2,T-1}, S_{2,T-2}) & \operatorname{cov}(S_{2,T-1}, W_{T-1}) \end{pmatrix}$$

$$\mathbf{C}_S = \begin{pmatrix} \operatorname{var}(S_{1,T-2}) & \operatorname{cov}(S_{1,T-2}, S_{2,T-2}) & \operatorname{cov}(S_{1,T-2}, W_{T-1}) \\ \operatorname{cov}(S_{2,T-2}, S_{1,T-2}) & \operatorname{var}(S_{2,T-2}) & \operatorname{cov}(S_{2,T-2}, W_{T-1}) \\ \operatorname{cov}(W_{T-1}, S_{1,T-2}) & \operatorname{cov}(W_{T-1}, S_{2,T-2}) & \operatorname{var}(W_{T-1}) \end{pmatrix}$$

Differentiation of Equation (A2.4) with respect to $\mathbf{t}_{T-1}$ yields the optimal time investment at calendar time T – 1: $\mathbf{t}^*_{T-1}(\mathbf{S}_{T-2}, W_{T-1})$.

A.2.3. Third step: Calculation of the inner problem at calendar time T – 2 of decision problem (5)

The inner problem at calendar time T – 2 of our two-period decision problem reads

$$\underset{\mathbf{N}_{T-2}}{\operatorname{Max}}\ E\left\{-\frac{1}{\alpha}\cdot e^{-\alpha\cdot W_t}\middle|\mathbf{S}_{T-2}\right\} \tag{A2.5}$$

where W_T contains $\mathbf{N}^*_{T-1}(\mathbf{S}_{T-2}, \mathbf{S}_{T-1}, W_{T-1})$ and $\mathbf{t}^*_{T-1}(\mathbf{S}_{T-2}, W_{T-1})$.

Using the n dimensional multinormal density of stock prices at calendar time T – 1, the expected value of Equation (A2.5) can be written as

$$-\frac{1}{\alpha}\cdot\int_{-\infty}^{\infty}\cdots\int_{-\infty}^{\infty}\frac{1}{\sqrt{\det\left(2\cdot\pi\cdot\mathbf{C}_{\mathbf{P}_T|\mathbf{S}_{T-2}}\right)}} \tag{A2.6}$$

$$\cdot\exp\left(-\frac{1}{2}\cdot\left(\mathbf{p}_{T-1} - \mathbf{E}_{\mathbf{P}_{T-1}|\mathbf{S}_{T-2}}\right)'\mathbf{C}^{-1}_{\mathbf{P}_{T-1}|\mathbf{S}_{T-2}}\left(\mathbf{p}_{T-1} - \mathbf{E}_{\mathbf{P}_{T-1}|\mathbf{S}_{T-2}}\right)\right)$$

$$\cdot e^{-\alpha\cdot W_T}\,d\,p_{1,T-1}\cdots d\,p_{n,T-1}$$

where $d\,p_{i,T-1}$ denotes the realization of the random variable price of stock i at calendar time T – 1 and $\mathbf{p}_{T-1}$ is the vector of the realization of stock prices at calendar time T – 1,

with

$$\mathbf{E}_{\mathbf{P}_{T-1}|\mathbf{S}_{T-2}} = E\{\mathbf{P}_{T-1}\} + \mathbf{COV}_{P_{T-1}S}(\mathbf{t}_{T-2})\mathbf{C}_S^{-1}(\mathbf{S} - E\{\mathbf{S}\})$$

$$\mathbf{C}_{\mathbf{P}_{T-1}|\mathbf{S}_{T-2}} = \mathbf{C}_{P_{T-1}} - \mathbf{COV}_{P_{T-1}S}(\mathbf{t}_{T-2})\mathbf{C}_S^{-1}\mathbf{COV}'_{P_{T-1}S}(\mathbf{t}_{T-2})$$

and

$$\mathbf{S}' = \begin{pmatrix} S_{1,T-2} & S_{2,T-2} \end{pmatrix}$$

$$\mathbf{COV}_{P_{T-1},S}(\mathbf{t}_{T-2}) = \begin{pmatrix} \operatorname{cov}(P_{1,T-1}, S_{1,T-2})(\mathbf{t}_{T-2}) & \operatorname{cov}(P_{1,T-1}, S_{2,T-2})(\mathbf{t}_{T-2}) \\ \operatorname{cov}(P_{2T-1}, S_{1,T-2})(\mathbf{t}_{T-2}) & \operatorname{cov}(P_{2,T-1}, S_{2,T-2})(\mathbf{t}_{T-2}) \end{pmatrix}$$

$$\mathbf{C}_S = \begin{pmatrix} \operatorname{var}(S_{1,T-2}) & \operatorname{cov}(S_{1,T-2}, S_{2,T-2}) \\ \operatorname{cov}(S_{2,T-2}, S_{1,T-2}) & \operatorname{var}(S_{2,T-2}) \end{pmatrix}$$

Differentiation of Equation (A2.6) with respect to $\mathbf{N}_{T-2}$ yields the optimal portfolio numbers at calendar time T – 2: $\mathbf{N}^*_{T-2}(\mathbf{S}_{T-2})$.

A.2.4. Fourth step: Calculation of the outer problem at calendar time T – 2 of decision problem (5)

The outer problem at calendar time T – 2 of our two-period decision problem reads

$$\underset{\mathbf{t}_{T-2}}{\text{Max}}\ E\left\{-\frac{1}{\alpha}\cdot e^{-\alpha \cdot W_T}\right\} \tag{A2.7}$$

where W_T contains $\mathbf{N}^*_{T-1}(\mathbf{S}_{T-2}, \mathbf{S}_{T-1}, W_{T-1})$, $\mathbf{t}^*_{T-1}(\mathbf{S}_{T-2}, W_{T-1})$, and $\mathbf{N}^*_{T-2}(\mathbf{S}_{T-2})$.

Using the m dimensional multinormal density of signals at calendar time T – 2, the expected value of Equation (A2.7) can be written as

$$-\frac{1}{\alpha}\cdot\int_{-\infty}^{\infty}\cdots\int_{-\infty}^{\infty}\frac{1}{\sqrt{\det(2\cdot\pi\cdot\mathbf{C}_{\mathbf{S}_{T-2}})}}\cdot\exp\left(-\frac{1}{2}\cdot(\mathbf{s}_{T-2}-\mathbf{E}_{\mathbf{S}_{T-2}})'\mathbf{C}^{-1}_{\mathbf{S}_{T-2}}(\mathbf{s}_{T-2}-\mathbf{E}_{\mathbf{S}_{T-2}})\right) \tag{A2.8}$$

$$\cdot\, e^{-\alpha \cdot W_T}\, d\,s_{1,T-2}\cdots d\,s_{m,T-2}$$

where $d\,s_{i,T-2}$ denotes the realization of the random signal S_i at calendar time T – 2 and $\mathbf{s}_{T-2}$ is the vector of the realization of signals at calendar time T – 2,

with

$$\mathbf{S}' = \left(S_{1,T-2} \quad S_{2,T-2}\right)$$

Differentiation of Equation (A2.8) with respect to $\mathbf{S}_{T-2}$ yields the optimal time investment at calendar time T – 2: $\mathbf{t}^*_{T-2}$.

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