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# Non-Negativity of Nominal and Real Riskless Rates, Arbitrage Theory, and the Null-Alternative Cash

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## Abstract

Pragmatic-world nominal riskless rates are non-negative. However, conventional arbitrage theory has yet to develop a theoretical justification of this phenomenon. – We define the null-alternative cash as an investor holding onto cash and refraining from investment and consumption ("doing nothing"); we use the null-alternative cash to prove that both nominal spot and nominal forward rates are non-negative and that prices of zero-coupon bonds do not increase with increasing maturity. In a positive inflation environment, however, both real spot and real forward rates might well become negative, but prices of zero-coupon bonds still do not increase with increasing maturity.



Key words: arbitrage theory, inflation, non-negativity of spot and forward rates, short selling constraints

*JEL classification*: G10, G12

# Non-Negativity of Nominal and Real Riskless Rates, Arbitrage Theory, and the Null-Alternative Cash

## 1. Introduction

It is well-known that pragmatic-world nominal riskless rates are non-negative as, for example, the yield curve of the European Central Bank for each EU Member States' government bonds or the German term structure computed by the Deutsche Bundesbank illustrate every month. However, conventional arbitrage theory has yet to provide a theoretical explanation for this phenomenon. Consequently, arbitrage-theoretical interest rate models do not question or explain non-negative riskless rates. Instead they simply develop models that keep nominal riskless rates positive. The classical example of this is the square root model of Cox/Ingersoll/Ross (1985), and a more contemporary example is the so-called "potential approach" which, for example, is outlined in Cairns (2004, pp. 131). Recently, the topic of justifying the non-negativity of nominal riskless rates has received some attention (see Schäfer/Kruschwitz/Schwake, 1998, p. 45 and 131; Nietert/ Wilhelm, 2001, pp. 16; Cairns, 2004, p. 53; and Munk, 2004, p. 153). These authors argue that negative nominal riskless rates could be exploited by holding cash, i.e., by pursuing what is called "mattress arbitrage:" borrowing money at a negative riskless rate and putting the money under the mattress would obviously realize arbitrage gains.

Although the concept of a mattress arbitrage is intuitive, it is rather informal and based only on examples. Therefore, mattress arbitrage cannot adequately take into account the coexistence of cash and a riskless asset in arbitrage-free markets, nor can it elaborate on the consequences of cash on the sign (i.e., positive or negative) of nominal riskless rates in a systematic way. Finally, mattress arbitrage does not address the question of non-negativity of real rates.

Having these points in mind, the objective of this paper is twofold. First, it integrates the null-alternative cash into a broader arbitrage-theoretical framework. Second, it explores consequences of cash in arbitrage-free markets to spot rates, forward rates, and the relation of zero-coupon bond prices of different maturities, under both zero and positive inflation environments.

The remainder of the paper is organized as follows. In Section 2 we define the null-alternative cash, in Section 3 we analyze the non-negativity of nominal riskless rates, and in Section 4 the non-negativity of real riskless rates. Section 5 summarizes paper.

## 2. The Null-Alternative Cash and Arbitrage Theory

### *Institutional Description of the Null-Alternative Cash*

If there are only low quality investment and consumption opportunities available, investors can refrain from investing and consuming ("doing nothing"). The amount of money neither consumed nor invested is automatically transferred to the next period in form of cash. This situation represents the null-alternative for the investor. The null-alternative cash is therefore a riskless asset with a nominal riskless rate of zero (see Tobin, 1958, p. 67). Moreover, the null-alternative cash is an integral component of investors' opportunity sets along with the "usual" riskless and risky assets.

However, obviously, cash requires a short selling constraint because it is impossible to sell "doing nothing" short. For ease of presentation we assume that, in general, there are no further market frictions and, in particular, no short selling constraints exist for other assets (besides cash).

### *A Brief Digression on Arbitrage Theory under Short Selling Constraints*

Since the null-alternative cash is an asset subject to short selling constraints, its pricing influence must be explored with the help of arbitrage theory under short selling constraints. Nonetheless, the results of this particular arbitrage theory are by far less known than their counterparts on frictionless markets. Therefore, it is advisable to briefly review the most important results of arbitrage theory under short selling constraints.

First, under short selling constraints the set of attainable cash flows (attainable via forming and holding portfolios over time) is a convex cone rather than a linear space. However, the set of cash flows attainable by portfolios of assets that are not subject to short selling constraints is in fact a linear space (see Jouini/Kallal, 1995).

Second, under short selling constraints there are two classes of assets: dominated and dominant assets (see, e.g., Ross, 1978, p. 455; and Detemple/Murthy, 1997, p. 1157). Investors would like to sell dominated assets to construct an arbitrage; yet binding short selling constraints prevent them from doing so. Additionally, purchasing dominated assets cannot be reasonable because rational non-satiated decision makers will at best supply, but not demand such dominated assets. For that reason, dominated assets are highly unlikely to be actively traded on pragmatic-world financial markets.

Third, the distinction between dominant and dominated assets makes it plausible that short selling constraints cause a modification of pricing results (see, e.g., Garman/Ohlson, 1981; and Jouini/Kallal, 1995). Under (binding or not) short selling constraints there exists a positive and sublinear functional $\phi$ that assigns to any attainable cash flow a real number that constitutes a lower bound for the price of the asset under consideration. In other words, it does not need to be a transaction price, but just a price offer to either buy or sell. Furthermore, for any actively traded asset the functional $\phi$ reproduces its transaction price. Although $\phi$ is positive and sublinear, it is, in general, not necessarily linear. Only for assets (and portfolios thereof) that are not subject to (binding or not) short selling constraints, the functional $\phi$ has the feature of positivity, price reproduction, and linearity.

## 3. Consequences of the Null-Alternative Cash to Nominal Riskless Rates

### *Proof that Conventional Arbitrage Theory Cannot Assure Non-Negative Nominal Riskless Rates*

In this section we prove that conventional arbitrage theory is unable to justify non-negative nominal riskless rates by offering a counter example. Accordingly, the initial step is to examine the following market segment with one riskless asset $A_1$ and two risky assets ($A_i$ and $A_j$) in a one-period frame:

**Table 1. Payoffs and Prices on a Financial Market**

| | | states of the world | |
|---|---|---|---|
| asset | price | state 1 | state 2 |
| $A_1$ | $111\frac{1}{9}$ | 100 | 100 |
| $A_i$ | $108\frac{8}{9}$ | 110 | 90 |
| $A_j$ | $115\frac{5}{9}$ | 80 | 120 |

The price functional $\phi$ on the above frictionless market reads $\phi_{S_1} = \frac{4}{9}$ and $\phi_{S_2} = \frac{2}{3}$; it is linear and positive and therefore the market is free of arbitrage (first fundamental theorem of asset pricing, see, e.g., Dybvig/Ross, 1992, p. 44). However, a look at the nominal riskless rate implied by this market reveals its negativity: $r = \frac{1}{\phi_{S_1} + \phi_{S_2}} - 1$ = -10%. – As a by-product, this example sheds some “negative light” on a proposition of Prisman (1986, p. 547, proposition I) which explicitly states: no-arbitrage holds “if, and only if, there exists a finite positive r, such that” no risky portfolio dominates the nominal risk-free rate r.

The reason why conventional arbitrage theory fails to guarantee a non-negative nominal riskless rate is quite easy to understand. Arbitrage theory merely derives statements about the position of the nominal riskless rate relative to other asset prices, but not with respect to a potential bound at zero. All that "conventional" arbitrage theory is able to achieve is to justify that 1 + nominal riskless rate must be positive, i.e., the nominal riskless rate must be greater than –100 % (see, e.g., Dybvig/Ingersoll/Ross, 1996, p. 3). If this was not the case, there would exist an asset with a positive price that had a risk-free negative payoff, a fact that contradicts the positivity of the price functional. For the same reason, two riskless investment opportunities (an explicit riskless asset $A_1$ and an implicit or "synthetic" one, which can be synthesized using risky assets on complete markets, in the above case: assets $A_i$ and $A_j$ ) cannot rule out a negative nominal riskless rate. Arbitrage theory can just force the explicit and the implicit nominal riskless rate to coincide (see Ross, 1977, p. 191).

In summary, neither several explicit nor implicit riskless assets or, alternatively, conventional arbitrage theory can make a case for the non-negativity of nominal riskless rates. The only way to guarantee non-negative nominal riskless rates might be to fall back on the null-alternative cash that has not been discussed by conventional arbitrage theory.

### *Non-Negativity of Nominal Spot Rates*

To elaborate on the consequences of the null-alternative cash to nominal spot rates, cash is compared to a zero-coupon bond that offers a payoff at time t of $Z_{Z,t} = 1$ with an initial investment of $I_{Z,0} = \frac{1}{\left(1 + r_{0,t}^{nom}\right)^{t}}$, where $r_{0,t}^{nom}$ denotes the nominal spot rate for the period between time 0 and t.

Assuming the zero-coupon bond is dominant, the price $P_{Z,0}$ at time 0 must, for no-arbitrage reasons, be:

$$P_{Z,0} = \phi_{0,t}(1) = \frac{1}{\left(1 + r_{0,t}^{nom}\right)^t} \tag{1a}$$

where $\phi_{0,t}$ denotes a price functional that translates payoffs at time t in arbitrage-free markets into prices at time 0.

The null-alternative cash (with payoff $Z_{Z,t} = 1$ and initial investment $I_{0,0} = 1$) has a lower price bound (due to arbitrage theory) of:

$$\frac{1}{\left(1 + r_{0,t}^{nom}\right)^t} = \phi_{0,t}(1) \leq 1 = I_{0,0} \tag{2a}$$

If inequality (2a) holds, cash will be dominated.

Now consider the reverse case: the null-alternative cash is dominant and the zero-coupon bond is dominated. In that event, arbitrage theory calls for the price $P_{0,0}$ at time 0 of cash to be:

$$P_{0,0} = \phi_{0,t}(1) = 1 \tag{1b}$$

and the zero-coupon bond has a lower price bound at time 0 of:

$$\phi_{0,t}(1) = 1 \leq \frac{1}{\left(1 + r_{0,t}^{nom}\right)^t} = I_{Z,0} \tag{2b}$$

Formulas (1a) and (2a) as well as (1b) and (2b) contain the information needed to clarify the influence of the null-alternative cash on nominal spot rates. When the zero-coupon bond is dominant, it must have a smaller price than the null-alternative cash despite an identical payoff (as formulas (1a) and (1b) demonstrate). However, this will only be true if the nominal spot rate $r_{0,t}^{nom}$ is positive. Moreover, a positive nominal spot rate does not permit an arbitrage with cash (i.e., so it does not violate the postulate of no-arbitrage) since cash cannot be sold short and, therefore, is not actively traded. – To make the zero-coupon bond dominated, it must have a higher price than the null-alternative cash; this can be achieved only by assuming a negative nominal spot rate $r_{0,t}^{nom}$. Finally, if the nominal spot rate $r_{0,t}^{nom}$ equals zero, the prices of the null-alternative cash and the zero-coupon bond must coincide making both assets candidates for active trading.

Two further conclusions can be drawn from the non-negativity of nominal spot rates. First, zero-coupon bond prices $\frac{1}{\left(1+r_{0,t}^{nom}\right)^{t}}$ must not exceed one. Second, all zero-coupon bonds with maturities less than infinity must have a positive price, i.e., $\frac{1}{\left(1+r_{0,t}^{nom}\right)^{t}} > 0$.

### *Non-Negativity of (Implied) Nominal Forward Rates*

The influence of the null-alternative cash on nominal forward rates will become apparent if cash is compared to an (implied) forward investment, i.e., the simultaneous sale and purchase of zero-coupon bonds at time 0 with maturities t and t + τ and payoffs of $Z_{Z,t} = -1$ and $Z_{Z,t+\tau} = 1$ respectively. If both zero-coupon bonds are actively traded, the price of the transaction in zero-coupon bonds in arbitrage-free markets is:

$$\phi_{0,t}(-1) + \phi_{0,t+\tau}(1) = -\frac{1}{\left(1+r_{0,t}^{nom}\right)^{t}} + \frac{1}{\left(1+r_{0,t+\tau}^{nom}\right)^{t+\tau}} \tag{3}$$

An initial investment of $I_{0,t} = 1$ in the null-alternative cash at time t yields a payoff of $Z_{0,t+1} = 1$. Rolling over this investment from time t + 1 until time t + τ induces an aggregated payoff of $Z_{0,t} = -1$ and $Z_{0,t+\tau} = 1$, and a cash flow of zero at every other time. Since the null-alternative cash is subject to short selling constraints, its "price" at time 0 has a lower bound of

$$I_{0,0} = 0 \geq \phi_{0,t}(-1) + \phi_{0,t+\tau}(1) = -\frac{1}{\left(1+r_{0,t}^{nom}\right)^{t}} + \frac{1}{\left(1+r_{0,t+\tau}^{nom}\right)^{t+\tau}} \tag{4}$$

Rearranging (4) obtains

$$\frac{1}{\left(1+r_{0,t}^{nom}\right)^{t}} \geq \frac{1}{\left(1+r_{0,t+\tau}^{nom}\right)^{t+\tau}} \tag{5}$$

or, more generally,

$$1 \geq \frac{1}{\left(1 + r_{0,1}^{nom}\right)^1} \geq \cdots \geq \frac{1}{\left(1 + r_{0,t}^{nom}\right)^t} \geq \cdots \geq \frac{1}{\left(1 + r_{0,T}^{nom}\right)^T} > 0 \tag{6}$$

The relation in (6) shows that zero-coupon bond prices do not increase with increasing maturity. That is, even if the term structure is inverse (e.g., the nominal spot rate $r_{0,T}^{nom}$ is much smaller than $r_{0,t}^{nom}$), no-arbitrage assures that the price of a zero-coupon bond with maturity T is not above the one with maturity t; the influence of discounting over a longer period dominates interest rate effects due to a possibly inverse term structure.

Additionally, relation (6) should not be confused with the results of Dybvig/Ingersoll/Ross (1996), who show that long zero-coupon rates for identical maturities, but at different times t and t + $\tau$ (with $\tau > 0$) cannot fall (i.e., $r_{t,T} \leq r_{t+\tau,T+\tau}$).

Moreover, since the quotient $\frac{\phi_{0,t+\tau}(1)}{\phi_{0,t}(1)} = \frac{\frac{1}{\left(1+r_{0,t+\tau}^{nom}\right)^{t+\tau}}}{\frac{1}{\left(1+r_{0,t}^{nom}\right)^{t}}}$ is an implied forward contract, we obtain

$$\frac{\phi_{0,t+\tau}(1)}{\phi_{0,t}(1)} = \frac{1}{\left(1+{}_0r_{t,t+\tau}^{nom}\right)^{\tau}} \tag{7}$$

where ${}_0r_{t,t+\tau}^{nom}$ denotes the (implied) nominal forward rate for the period between time t and t + $\tau$ for a contract that has been entered at time 0.

Given this relationship, it becomes immediately clear from (6) that

$$\frac{\phi_{0,t+\tau}(1)}{\phi_{0,t}(1)} = \frac{1}{\left(1+{}_0r_{t,t+\tau}^{nom}\right)^{\tau}} \leq 1 \tag{8}$$

must be true for zero-coupon bonds that are not dominated by cash. In other words, the nominal forward rate must be non-negative.

## 4. Nominal Versus Real Riskless Rates

The results on the non-negativity of riskless rates so far have been derived for nominal riskless rates. Therefore, it could be asked whether the three findings presented above, the non-negativity of nominal spot and nominal forward rates as well as the relation between prices of zero-coupon bonds and maturity, will still hold for real riskless rates under positive inflation. To keep the exposition simple, we look only at deterministic inflation rates.

Positive inflation means that the price of a consumption good at time t + 1 equals its price at time t multiplied by $1+i_{t+1}$, where $i_{t+1}$ is positive and denotes the inflation rate at time t + 1. This signifies money loses value, and nominal and real riskless rates are connected by the following definition:

$$\frac{\left(1+r_{0,t}^{nom}\right)^t}{\prod_{\theta=1}^{t}\left(1+i_\theta\right)} = \left(1+r_{0,t}^{real}\right)^t \tag{9}$$

Based on relations (9) and (1a) and according to arbitrage theory, the following relationship for the price of a dominant zero-coupon bond with maturity t must hold

$$P_{Z,0} = \phi_{0,t}(1) = \frac{1}{\left(1+r_{0,t}^{nom}\right)^t} = \frac{1}{\left(1+r_{0,t}^{real}\right)^t \cdot \prod_{\theta=1}^{t}\left(1+i_\theta\right)} \tag{10}$$

and the lower price bound of the dominated asset cash must be

$$\frac{1}{\left(1+r_{0,t}^{real}\right)^t \cdot \prod_{\theta=1}^{t}\left(1+i_\theta\right)} = \phi_{0,t}(1) \le \frac{1}{\left(1+r_{0,t}^{real\ cash}\right)^t \cdot \prod_{\theta=1}^{t}\left(1+i_\theta\right)} = \frac{1}{\left(1+r_{0,t}^{nom\ cash}\right)^t} = 1 = I_{0,0} \tag{11}$$

Since the nominal rate on cash $r_{0,t}^{nom\ cash}$ equals zero, we gain

$$\frac{1}{\left(1+r_{0,t}^{real}\right)^t \cdot \prod_{\theta=1}^{t}\left(1+i_\theta\right)} \le 1 \tag{12}$$

from which follows

$$r_{0,t}^{real} \geq \sqrt[t]{\frac{1}{\prod_{\theta=1}^{t}(1+i_\theta)}}-1 \tag{13}$$

Relation (13) demonstrates that real spot rates can indeed be negative in a positive inflation environment. This theoretical result is in perfect alignment with the empirical findings of, for example, Fisher (1977, p. 44).

To further illustrate the relation shown in equation (13), let us examine the real spot rate between time 0 and time 1:

$$r_{0,1}^{real} \geq \frac{1}{1+i_1}-1=-\frac{i_1}{1+i_1} \tag{14}$$

Relation (14) clarifies that the one-period real spot rate must be slightly larger than the negative inflation rate for this period.

Although real spot rates can be negative, nominal spot rates still have to be positive to dominate the null-alternative cash. This means that relation (6) remains valid and zero-coupon bond prices do not increase with increasing maturity:

$$\begin{aligned} 1 &\geq \frac{1}{\left(1+r_{0,t}^{real}\right)\cdot\left(1+i_1\right)} \geq \cdots \geq \frac{1}{\left(1+r_{0,t}^{real}\right)^t \cdot \prod_{\theta=1}^{t}\left(1+i_\theta\right)} \geq \\ &\cdots \geq \frac{1}{\left(1+r_{0,t}^{real}\right)^T \cdot \prod_{\theta=1}^{T}\left(1+i_\theta\right)} > 0 \end{aligned} \tag{15}$$

Finally, using (9) to express the nominal rate in (8), we obtain

$$\frac{\phi_{0,t+\tau}(1)}{\phi_{0,t}(1)}=\frac{1}{\left(1+{}_0r_{t,t+\tau}^{nom}\right)^\tau}=\frac{1}{\left(1+{}_0r_{t,t+\tau}^{real}\right)^\tau \cdot \prod_{\theta=t}^{t+\tau}\left(1+i_\theta\right)} \leq 1 \tag{16}$$

According to relation (16) real (implied) forward rates can be negative. To be more precise, it must hold for (implied) real forward rates

$$ {}_{0}r^{\text{real}}_{t,t+\tau} \geq \sqrt[\tau]{\frac{1}{\prod_{\theta=t}^{t+\tau}\left(1+i_{\theta}\right)}}-1 \tag{17} $$

The economic reason behind the potential negativity of real spot and forward rates is the following: In a zero-inflation environment, the null-alternative cash offers a zero nominal interest rate which constitutes a lower bound for nominal rates. Under positive inflation, however, there does not exist a corresponding asset that offers a zero-real rate. One possible hedge against inflation would be a bundle of consumption goods. These goods, however, do not, first, constitute a perfect hedge, and, second, are not as liquid as financial assets and, thus, cannot prevent real rates from becoming negative; namely illiquidity prevents arbitrage transactions from working properly. Another potential hedge might be inflation protected bonds. However, there are not many differing maturities available (see Roll, 2004, p. 32) and the market for these bonds is less liquid than the market for conventional bonds (see Kothari/Shanken, 2004, p. 67). For these reasons, real rates can nevertheless become negative as Kothari/Shanken (2004, p. 58) demonstrate.

## 5. Conclusion

This paper used the null-alternative cash, i.e., “doing nothing”, to prove two empirical phenomena that conventional arbitrage theory has failed to address: first, that nominal spot and nominal forward rate are non-negative, and, second, that zero-coupon bond prices do not increase with increasing maturity. In a positive inflation environment, however, both real spot and real forward rates might well become negative, but prices of zero-coupon bonds still do not increase with increasing maturity.

These results may contain useful information for the identification of arbitrage opportunities in the pragmatic world. Transactions in bond markets are not subject to estimation risk with respect to future payoffs as long as bonds are seen as default free (e.g., government bonds). Therefore, the obeservation of negative spot or forward rates allows for the recognition of arbitrage profits in a narrower sense versus only recognizing profit potential with a positive mean (so-called statistical arbitrage, see Bondarenko, 2003) as it is often the case with arbitrage transactions involving risky assets.

**References**


Bondarenko, O. (2003) Statistical Arbitrage and Securities Prices, *The Review of Financial Studies*, **16**, 875-919.

Cairns, A. J. G. (2004) *Interest Rate Models: An Introduction*, Princeton University Press, Princeton.

Cox, J. C, J. E. Ingersoll, Jr. and St. A Ross (1985) A Theory of the Term Structure of Interest Rates, *Econometrica*, **53**, 385-407.

Detemple, J. and S. Murthy (1997) Equilibrium Asset Prices and No-Arbitrage with Portfolio Constraints, *The Review of Financial Studies*, **10**, 1133-1174.

Deutsche Bundesbank statistics section, *http://www.bundesbank.de/statistik/statistic.en.php*, (accessed on June 6, 2004).

Dybvig, Ph. H., J. E. Ingersoll, Jr. and St. A. Ross (1996) Long Forward and Zero-Coupon Rates Can Never Fall, *Journal of Business*, **69**, 1-25.

Dybvig, Ph. H. and St. A. Ross (1992) Arbitrage, in Palgrave, R. H. and P. Newman, Eds. *The New Palgrave Dictionary of Money & Finance A – I*, Stockton, New York, 43-50.

European Central Bank statistics section, *http://www.ecb.int/stats/stats.htm* (accessed on June 6, 2004).

Fisher, I. (1977) *The Theory of Interest*, reprint, Porcupine Press, Philadelphia.

Garman, M. B. and J. A. Ohlson, J. A. (1981) Valuation of Risky Assets in Arbitrage-free Economies with Transactions Costs, *Journal of Financial Economics*, **9**, 271-280.

Jouini, E. and H. Kallal (1995) Arbitrage in Securities Markets with Short-Sales Constraints, *Mathematical Finance*, **5**, 197-232.

Kothari, S. P. and Shanken, J. (2004) Asset Allocation with Inflation-Protected Bonds, *Financial Analysts Journal*, (Jan/Feb) **60**, 54-70.

Munk, C. (2004) *Fixed Income Analysis: Securities, Pricing and Risk Management*, Lecture Note, Department of Accounting and Finance, University of Southern Denmark, http://www.sam.sdu.dk/undervis/92220.F04/#Udervisningsmatriale.

Nietert, B. and J. Wilhelm (2001) Some Economic Remarks on Arbitrage Theory, *Working Paper*, Passau University, http://ssrn.com/abstract=260008.

Prisman, E. Z. (1986) Valuation of Risky Assets in Arbitrage Free Economies with Frictions, *The Journal of Finance*, **41**, 545-560.

Roll, R. (2004) Empirical TIPS, *Financial Analysts Journal*, (Jan/Feb) **60**, 31-53.

Ross, St. A. (1977) Return, Risk and Arbitrage, in Friend, I. and J. Bicksler, Eds. *Risk and Return in Finance*, Ballinger, Cambridge, 189-218.

Ross, St. A. (1978) A Simple Approach to the Valuation of Risky Streams, *Journal of Business*, **51**, 453-475.

Schäfer, D., L. Kruschwitz and M. Schwake (1998) *Studienbuch Finanzierung und Investition*, Second Edition, Oldenbourg, München et al.

Tobin, J. (1958) Liquidity Preference as Behavior Towards Risk, *The Review of Economic Studies*, **25,** 65-86.