Local and spillover shocks in implied market volatility: evidence for the U.S. and Germany

Niklas Wagner* , Alexander Szimayer

Center for Entrepreneurial and Financial Studies, Munich University of Technology, 80290 Munich, Germany
Department of Accounting and Finance, The University of Western Australia, Crawley, WA 6009, Australia

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Abstract

The occurrence and the transmission of large shocks in international equity markets is of essential interest to the study of market integration and financial crises. To this aim, implied market volatility allows to monitor ex ante risk expectations in different markets. We investigate the behavior of implied market volatility indices for the U.S. and Germany under a straightforward mean reversion model that allows for Poisson jumps. Our empirical findings for daily data in the period 1992 to 2002 provide evidence of significant positive jumps, i.e. situations of market stress with positive unexpected changes in ex ante risk assessments. Jump events are mostly country-specific with some evidence of volatility spillover. Analysis of public information around jump dates indicates two basic categories of events. First, crisis events occurring under spillover shocks. Second, information release events which include three subcategories, namely—worries about as well as actual—unexpected releases concerning U.S. monetary policy, macroeconomic data and corporate profits. Additionally, foreign exchange market movements may cause volatility shocks.

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1. Introduction

The occurrence and the transmission of large shocks between international equity markets is of essential interest to the study of market integration, the occurrence of periods of market
stress and the analysis of financial crises. While most empirical studies in the area are based on prices, it appears promising to study price volatility as well. Option pricing theory implies that options markets reject market participants’ expectations of future asset price volatility. The theory offers implied market volatility as a risk measure which not only reflects ex ante risk expectations but also has an immediate impact on traded option prices. Therefore, shocks in implied volatility are crucial with respect to stock market and option market price formation and to the hedging of derivative securities. Shocks to implied volatility can be modeled as jumps, i.e. as time series discontinuities; see for example the models by Duffie et al. (2000) and Duan et al. (2003). Recent empirical investigations on jumps in stock market volatility include Andersen et al. (2003), Duan et al. (2003), Eraker et al. (2003) which are based on models of unobservable volatility and Wagner and Szimayer (2001) with a study based on implied volatility.

The present paper contributes to the existing literature on implied equity market volatility by examining jump risk. We propose a straightforward mean reverting model which allows for Poisson jumps in implied volatility; this formally relates to the Merton (1976) model for stock prices. Whereas jump risk in stock prices is typically considered as unsystematic, jumps in market volatility represent a source of systematic risk. The analysis of implied volatility for the U.S. and Germany is first with respect to the univariate dynamics, then with a focus on periods of market stress and on spillover jump events.

Jump risk in implied market volatility is of interest for several reasons. First, it is relevant to examining volatility shocks in—and the transmission between—international equity markets. Apart from volatility spillover under normal market conditions (see e.g. Hamao et al. (1990) and Susmel and Engle (1994)) one may examine events of market stress and consider spillover conditional on stress in one market. As it is known, changes in volatility have played a major role in many of the recent cases of financial distress. Second, whereas derivatives hedging is achievable under smooth price and volatility movements, it will seriously be challenged by abrupt changes in market volatility. Learning more about such jumps can help us to explain market participants behavior. As such—in order to avoid jump risk (or Vega risk as it is frequently called)—many traders tend to close their positions in advance of certain public news releases. It would be of interest to empirically detect the relevant events and thereby separate between predictable, information-release driven, and unpredictable events. Previous empirical research in this direction was based on prices and for example carried out by Haugen et al. (1991) and Lobo (1999). Finally, implied market volatility is a key pricing factor for option portfolios which—in principal—may be hedged by volatility derivatives (see e.g. Grünbichler and Longstaff (1996), Locarek-Junge and Roth (1998) and Wagner and Szimayer (2001)) or by trading straddles (see e.g. Brenner et al. (2000)). The growing need for volatility derivatives clearly justifies a closer inspection of the dynamics of the underlying implied volatility.

We investigate the dynamic behavior of daily implied market volatility for the U.S. and Germany as measured by the VIX and the VDAX volatility indices during the years 1992 to 2002. Inference is based on method of moments estimation and is robust with respect to effects of autocorrelation and heteroskedasticity. Our findings show that—given a difference in the time horizons of the indices—the univariate parameters indicate comparable behavior
for both indices, where a longer horizon implies a higher variability. In particular, the results provide evidence of significant positive jumps for both indices. We find that these are more frequently observed for the VDAX having a nearly twice as large estimated jump intensity as compared to the VIX. Furthermore, jump events are mostly country-specific, i.e. we find weak evidence of market spillover under market stress. Estimating in sample jump dates allows us to study how the observed shocks to ex ante market risk expectations relate to public news. The results indicate two basic categories of events. First, major crisis events occurring under spillover shocks. Second, information release events which include three subcategories, namely—worries about as well as actual—unexpected releases concerning (i) U.S. monetary policy, (ii) macroeconomic data and (iii) corporate profit reports or single profit warnings. Additionally, there is evidence that foreign exchange market movements may cause volatility shocks due to worries about prevailing exchange rate relations.

The remainder of the paper is organized as follows. A review of related literature on implied volatility and implied volatility indices is given in the following section. Sections 3 and 4 describe the mean reverting jump diffusion model and the estimation methodology. A simple hypothesis test for jump events is outlined in Section 5. Section 6 contains the empirical results on model estimation and jump event public news releases. The paper concludes with a brief outlook in Section 7.

2. Related literature on implied volatility

Research on stochastic implied volatility has been tremendous during the last three decades. The calculation of implied volatility typically requires a model of option prices, the model inputs (except for volatility) and observable option prices. Starting with the proposal by Latané and Rendleman (1976), the most common procedure is to assume the Black/Scholes constant variance option pricing model with inputs given by observable market variables. Furthermore, the prices of liquid at the money options are used, where the option value is approximately a linear function of the stock price. Implied volatility can be interpreted as the market participants’ ex ante assessment of risk over a given future time period; see e.g. Mayhew (1995) for a literature survey.

Discussions on implied volatility mainly focus on (i) the methodology of inferring estimates, on (ii) obvious deviations from the assumption of constant variance and on (iii) the information content of implied volatility. Methodological aspects have been discussed for example by Corrado and Miller (1996) who describe a method for efficiently weighting the volatility estimates from different strikes and maturities. Hentschel (2003) considers the effects of noise in the input parameters which yield implied volatility and suggests a less error-sensitive estimation technique. Lamoureux and Lastrapes (1993) use the Hull/White stochastic volatility model to infer estimates of implied volatility which provides evidence against the joint hypothesis that options markets are informationally efficient and that option prices are explained by the Hull/White model which assumes that volatility risk is unpriced. The informational content of implied volatility for predicting future price volatility is mostly found to dominate that of historical information; see e.g. Day and Lewis (1992), Canina and Figlewski (1993), Fleming et al. (1995) and Claessen and Mittnik (2002), among others.
Despite these discussion points, implied volatility is undoubtedly an important market-determined measure of short term expected risk. Due to its importance, several indices of implied stock market volatility have been constructed since 1993.¹ These indices include for example the CBOE Market Volatility Index (VIX) and the NASDAQ Volatility Index (VXN). European indices exist for France (VX1), Germany (VDAX) and Greece (GVIX). There is a recently growing literature on the behavior and the interrelations of implied volatility as measured by volatility indices. These include for example Whaley (1993, 2000), Aboura and Villa (1999), Gemmill and Kamiyama (2000), Giot (2003) and Skiadopoulos (2003). A positive jump component in volatility was examined in a few other empirical studies so far. Based on the VDAX volatility index, Wagner and Szimayer (2001) document a statistically significant positive jump component.²

In this study we focus on the two earliest volatility indices, namely the VIX and the VDAX which were established in February 1993 and December 1994, respectively. The calculation of both indices is based on call and put index option prices. Implied volatilities are calculated using at the money options with two different strikes and times to maturity. Differences in volatility index construction arise from differences in the underlying stock index options. Since EUREX index options on the DAX are European-style and dividends are reinvested in the underlying index, the Black/Scholes pricing formula can be applied, whereas American-style OEX options with dividend payments on the S&P 100 require the binomial method to compute implied volatilities for the VIX.³ The underlying DAX index contains 30 stocks, whereas the S&P 100 represents a portfolio of 100 stocks. The weighting scheme of the VIX implies taking the arithmetic mean of put and call volatilities, calculating moneyness-weighted volatilities for different maturities and then taking the time-weighted mean of implied variances in order to obtain a constant time to maturity of 22 trading-days (equal to 30 calendar-days). The VDAX weighting scheme aggregates call and put volatilities of different strikes by fitting an OLS regression on model versus observed option prices. As with the VIX, a constant time to maturity is maintained by calculating the time-weighted mean of implied variances, where VDAX time to maturity is equal to 45 calendar-days. Without going into further details, one can conclude that apart from the weighting scheme, the main difference between the two volatility indices is their time to maturity.⁴

¹ Early proposals on volatility indices forming an underlying for trading volatility go back to Gastineau (1977) and Cox and Rubinstein (1985, A8).
² There is also recent evidence of positive jumps in unobserved volatility. Duffie et al. (2000) develop a bivariate affine model of stock prices and volatility jointly driven by a diffusion as well as a jump process. Given this methodological setting, Eraker et al. (2003) provide empirical evidence on volatility jumps and conclude that return models without a jump component in volatility suffer from mis-specification. Duan et al. (2003) find empirically that the incorporation of jumps in volatility adds significantly to the description of S&P 500 index returns.
³ In September 2003, CBOE announced a new VIX which is now based on S&P 500 index options. For reasons of continuity we conduct our study based on the old VIX. Using the new VIX would not affect the basic results of our empirical study.
⁴ For more detailed information on the indices refer to the respective CBOE and EUREX stock exchange websites. Also, Whaley (1993) and Fleming et al. (1995) give a description of the VIX and Wagner and Szimayer (2001) give one of the VDAX.
3. The volatility model

Considering the empirical results by Stein (1989) among others, it has become common practice to model stochastic volatility as a stationary mean reverting process. A natural extension to this approach is to capture the possibility of discontinuous sample paths in the volatility process.\(^5\) This can be done with a simplified version of the Merton (1976) jump diffusion framework.

Assume that within a frictionless market volatility \(V_t > 0\) evolves continuously in time \(t\), governed by a stochastic differential equation of the form:

\[
dV_t = \alpha(L - V_t)\,dt + \sigma V_t\,dB_t + \kappa V_t\,dN_t.
\]

This equation defines a mean reverting Poisson jump diffusion process. The parameters \(\alpha\), \(L\), \(\sigma\) and \(\kappa\) are constant, denoting the instantaneous mean reversion rate, the mean level, the scale parameter and the non-negative jump height \(\kappa \geq 0\), respectively. The increment \(dB_t\) represents Brownian motion and \(dN_t\) corresponds to a homogeneous Poisson process \(N_t\) with intensity \(\lambda > 0\) per unit time. The processes \(B_t\) and \(N_t\) are assumed to be stochastically independent. Since \(E(dB_t) = 0\) and \(E(dN_t) = \lambda \,dt\), the expected instantaneous change in volatility is:

\[
E(dV_t) = \alpha(L - E(V_t))\,dt + \kappa \lambda E(V_t)\,dt.
\]

Setting the expected change in \(V_t\) equal to zero, yields the solution for the long run stationary volatility expectation: \(^6\)

\[
\lim_{t \to \infty} E(V_{t-}) = \lim_{t \to \infty} E(V_t) = \frac{\alpha}{\alpha - \kappa \lambda} L.
\]

Hence, assuming a strictly positive volatility with a strictly positive mean reversion coefficient \(\alpha > 0\) and a strictly positive jump component with \(0 < \kappa \lambda < \alpha\), the long run expected volatility exceeds the mean reversion level \(L\). If \(\kappa = 0\), the model reduces to a mean reverting diffusion and the long run expected volatility is equal to the mean reversion level.

4. Discrete time estimation methodology

This section explains our econometric methodology which is based on an Euler-type discrete time version of the continuous-time process and methods of moments estimation. Although the approach suffers from various shortcomings pointed out in the recent literature, we prefer it as a simple and robust method of inference for our application. Other approaches would involve non-parametric or simulated maximum likelihood estimation, for example.

\(^5\) Fleming et al. (1995, p. 272) point out that there are several spikes in their historical VIX time series. Bates (2000, p. 219) argues that large increases observed in volatility put doubt on the pure diffusion assumption and suggests a jump process component.

\(^6\) In the limit as \(t\) approaches infinity, expected volatility before and after a possible jump event is identical.
Eq. (1) can only be observed at discrete points of time. A corresponding discrete time formulation is:

$$\Delta V_t = \alpha(L - V_{t-1}) \Delta t + \sigma \sqrt{\Delta t} V_{t-1} \epsilon_t + \kappa V_{t-1} q_t,$$

where $\epsilon_t \sim N(0; 1)$ and $q_t \sim P(\lambda \Delta t)$. Defining the relative change in volatility as a random variable $Y_t$, we can write:

$$Y_t \equiv \Delta V_t / V_{t-1} = \left( \kappa \lambda - \alpha + \frac{\alpha L}{V_{t-1}} \right) \Delta t + \sigma \sqrt{\Delta t} \epsilon_t + \kappa V_{t-1} q_t - \lambda \Delta t).$$

(4)

Eq. (4) is used for inferring model parameter estimates by the method of moments. This method exploits the fact that, under the hypothesis stating that $\Delta V_t$ is distributed according to model Eq. (3), observed sample moments converge to the corresponding theoretical moments. The theoretical moments can be expressed as a function of the parameter vector $\theta = (\alpha, L, \sigma^2, \kappa, \lambda)$ and the volatility level of the preceding period $V_{t-1}$. The sample moments are calculated as the arithmetic mean of the corresponding moment function, where the Slutsky-theorem provides consistency of the resulting parameter estimates. For the methodology refer to Greene (2003), for example.

Ignoring terms of order $O(\Delta t^2)$ and higher and setting $\Delta t = 1$—given the realized observations $y_t$ and $v_{t-1}$, $t = 1, \ldots, T$—parameter estimates can be derived by equating the following vector to zero:

$$g_T(\theta, v_t) = 1/T \sum_{t=1}^{T} \begin{pmatrix} y_t v_{t-1} - [(\kappa \lambda - \alpha) v_{t-1} + \alpha L] \\ y_t - [\kappa \lambda - \alpha + \alpha L v_{t-1}] \\ y_t^2 - [\kappa^2 \lambda + \sigma^2] \\ y_t^3 - [\kappa^3 \lambda] \\ y_t^4 - [\kappa^4 \lambda] \end{pmatrix}.$$  

(5)

The sample moments in Eq. (5) are the mean change in daily volatility, $m_0 = 1/T \sum y_t v_{t-1}$, and the first four moments of the relative daily volatility changes, $m_k = 1/T \sum y_t^k$, $k = 1, \ldots, 4$. Solving $g_T(\theta, v_t) = 0$ yields a parameter vector estimate $\hat{\theta}$ which is asymptotically normal; see Hansen (1982).

5. Jump events in discrete time

Starting with the discrete time definition of the relative change in volatility according to Eq. (4), now focus on the stochastic jump diffusion component. The model error term can be expressed as:

$$\eta_t = \sigma \sqrt{\Delta t} \epsilon_t + \kappa (q_t - \lambda \Delta t).$$

(6)

As $\epsilon_t$ and $q_t$ are both i.i.d. and independent, with a constant time increment $\Delta t$, it follows that the error term is distributed as a white noise process: $\eta_t \sim WN [0; (\sigma^2 + \kappa^2 \lambda) \Delta t]$. Setting $\Delta t = 1$ in Eq. (6) results in the linear approximation:

$$\eta_t \approx \sigma \epsilon_t + \kappa (p_t - \lambda).$$

(7)
where \( p_t \) is drawn from a discrete distribution with \( P(p_t = 1) = \lambda \) and \( P(p_t = 0) = 1 - \lambda \). Dividing both sides of Eq. (7) by the scale parameter \( \sigma \) yields the standardized error term:

\[
\xi_t = \epsilon_t + \rho(p_t - \lambda), \quad (8)
\]

with the jump-to-diffusion ratio defined as: \( \rho \equiv \kappa/\sigma \).

Testing for a possible jump event occurring at time \( t \) implies testing the null hypothesis \( H_0: p_t = 0 \) against the alternative \( H_1: p_t = 1 \). In order to provide statistical evidence on this pair of hypotheses, we choose a constant threshold value \( b \in \mathbb{R} \). Let this threshold value separate between Brownian noise and Poisson jump events according to the following decision rule:

\[
\text{reject } H_0 \iff (\xi_t > b).
\]

The resulting probabilities for a type I and a type II error are given by:

\[
e_{I}(b) = P(\xi_t > b | p_t = 0),
\]

\[
e_{II}(b) = P(\xi_t \leq b | p_t = 1).
\]

With independently distributed error terms \( \epsilon_t \) and \( p_t \) it follows from Eq. (8):

\[
e_{I}(b) = 1 - \Phi(b + \rho \lambda),
\]

\[
e_{II}(b) = \Phi(b - \rho + \rho \lambda).
\]

\( \Phi(x) \) denotes the cumulative standard normal probability density. We have:

\[
\frac{\partial e_{I}(b)}{\partial \rho} = -\phi(b + \rho \lambda)\lambda < 0 \quad \text{and} \quad \frac{\partial e_{II}(b)}{\partial \rho} = -\phi(b - \rho + \rho \lambda)(1 - \lambda) < 0
\]

with \( \phi(x) \equiv d\Phi(x)/dx \) and \( 0 < \lambda < 1 \). Note that a larger jump-to-diffusion ratio \( \rho \) reduces the error probabilities \( e_{I}(b) \) and \( e_{II}(b) \) and hence increases the power of the test. In hypothesis testing, it is common to set the type I error probability \( e_{I} \) to a fixed significance level. This gives the boundary value:

\[
b_{e_{I}} = \Phi^{-1}(1 - e_{I}) - \rho \lambda.
\]

Here, the type II error probability is then fixed with the choice of the significance level \( e_{I} \). It follows:

\[
e_{II}(b_{e_{I}}) = \Phi(\Phi^{-1}(1 - e_{I}) - \rho).
\]

6. Empirical results

In this part of the paper, the historical time series of the VIX and the VDAX volatility indices are examined. First, the focus is on the time-series properties of the index changes. Then, jump events are filtered based on the above methodology distinguishing local and spillover jump events. We finally address public news releases given at our estimated jump dates.
Table 1
Descriptive statistics for the daily VIX and VDAX changes in the sample period January 2, 1992 to December 31, 2002

<table>
<thead>
<tr>
<th>Sample statistic</th>
<th>VIX volatility index</th>
<th>VDAX volatility index</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta v_t$</td>
<td>0.00468</td>
<td>0.0121</td>
</tr>
<tr>
<td>Std ($\Delta v_t$)</td>
<td>1.487</td>
<td>1.213</td>
</tr>
<tr>
<td>Skew ($\Delta v_t$)</td>
<td>0.570</td>
<td>0.924</td>
</tr>
<tr>
<td>Kurt ($\Delta v_t$) $- 3$</td>
<td>-3.835</td>
<td>8.800</td>
</tr>
<tr>
<td>$\hat{\rho}(1)$</td>
<td>-0.105$^a$</td>
<td>-0.055$^a$</td>
</tr>
<tr>
<td>$\hat{\rho}(2)$</td>
<td>-0.064$^a$</td>
<td>-0.060$^a$</td>
</tr>
<tr>
<td>$\hat{\rho}(3)$</td>
<td>-0.066$^a$</td>
<td>-0.038$^a$</td>
</tr>
<tr>
<td>$Q(3)$</td>
<td>53.89$^a$</td>
<td>22.37$^a$</td>
</tr>
<tr>
<td>$Q(10)$</td>
<td>80.46$^a$</td>
<td>40.83$^a$</td>
</tr>
<tr>
<td># Observations</td>
<td>2771</td>
<td>2760</td>
</tr>
</tbody>
</table>

$^a$ Denotes an estimate which is significant at the 95% confidence level for a double-sided test.

In order to interpret the empirical results, recall that the DAX Index contains 30, whereas the S&P 100 Index contains a hundred stocks. This difference in the number of index constituents results in a lower volatility level for the US market proxy. However, since a vast amount of unsystematic risk will be diversified away even with only thirty stocks, we would expect differences in volatility levels to be mostly attributable to differences in market risk. Another important point concerns the difference in the ex ante horizon covered by the two indices. In the context of stochastic volatility, a term structure of implied volatilities will be observed: implied volatilities plotted against an increasing time to maturity of the options from which they are calculated will, in general, be upward or downward sloping. This was initially observed by Stein (1989), showing that mean reversion in implied volatility corresponds to a term structure in implied volatilities. As a consequence, relatively short time to expiration volatilities show larger variation across time than those calculated form options with longer maturities. Since VDAX time to maturity is 1.5 times larger than the one of the VIX, we would expect that total variation in VIX should be larger than variation in VDAX. 8

6.1. The VIX and VDAX sample

The chosen sample covers the time period beginning January 2, 1992, ending December 31, 2002. These eleven years of data contain 2772 daily observations for the VIX and 2761 daily observations for the VDAX. The levels of the two volatility indices are plotted in Fig. 1. As the plot indicates, both indices are highly dependent. Also, the general level of market volatility increased during 1996 and remained higher thereafter.

Descriptive statistics for the realized changes $\Delta v_t \equiv v_t - v_{t-1}$ in VIX and VDAX volatility are given in Table 1. The sample mean change is positive in both samples which

7 For empirical results see also Franks and Schwartz (1991) for stock index volatility and Xu and Taylor (1994) for foreign exchange volatility.

8 It is possible to adjust for this difference by scaling both indices to a common time to maturity. We do not further consider this issue here as standard scaling by the square-root-of-time is correct under normality only and hence only an approximation. Also, we want our empirical results to represent the properties of the original volatility indices.
indicates the positive trend in volatility. The sample standard deviation is higher for the VIX, which provides evidence for a larger variation in the VIX series. Sample skewness and excess kurtosis show that changes in the volatility indices are highly non-normal. Excess kurtosis is high and—jointly with positive skewness—points out the possibility of extreme positive daily index changes. Sample autocorrelation coefficients are significant for lag one, two and three for both index changes. The Ljung/Box statistics for a lag of 3 and 10 trading days, $Q(3)$ and $Q(10)$, show that the white noise hypothesis for VIX and VDAX changes can be rejected at high confidence levels. The time series dependence—i.e. persistence—is even stronger for the VIX.

6.2. Parameter estimation results

Based on the mean reverting jump diffusion model of Section 4, Table 2 provides a summary of the model parameter estimates $\theta = (\alpha, L, \sigma^2, \kappa, \lambda)$ for the VIX and the VDAX series. We report Newey and West (1987) heteroskedasticity and autocorrelation consistent standard errors. Thereby, the Newey and West (1994) approach serves as an optimal selection criterion for the time series lag used in the estimation of the asymptotic covariance matrix of the vector of moment conditions, $g_T(\theta, v_t)$. We additionally report the resulting estimates of the standard deviation $\hat{\sigma}$, the jump-to-diffusion ratio $\hat{\kappa}/\hat{\sigma}$, the expected long run volatility $\hat{\alpha} L/(\hat{\alpha} - \hat{\kappa} \hat{\lambda})$, and the total error variance $\hat{\sigma}^2 + \hat{\kappa}^2 \hat{\lambda}$. 
Table 2
Parameter estimates for the VIX and the VDAX volatility index in the sample period January 2, 1992 to December 31, 2002

<table>
<thead>
<tr>
<th>Parameter estimate</th>
<th>VIX volatility index</th>
<th>VDAX volatility index</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \hat{\alpha} )</td>
<td>0.01334 (5.12)(^a)</td>
<td>0.006708 (2.49)(^a)</td>
</tr>
<tr>
<td>( \hat{L} )</td>
<td>20.06 (16.88)(^a)</td>
<td>20.54 (10.21)(^a)</td>
</tr>
<tr>
<td>( \hat{\sigma}^2 )</td>
<td>0.003243 (16.50)(^a)</td>
<td>0.001805 (15.17)(^a)</td>
</tr>
<tr>
<td>( \hat{\lambda} )</td>
<td>0.001627 (2.42)(^a)</td>
<td>0.002947 (2.61)(^a)</td>
</tr>
<tr>
<td>( \hat{\kappa} )</td>
<td>0.5098 (11.94)(^a)</td>
<td>0.3514 (12.96)(^a)</td>
</tr>
<tr>
<td>( \hat{\sigma} = \sqrt{\hat{\sigma}^2} )</td>
<td>0.05695</td>
<td>0.04248</td>
</tr>
<tr>
<td>( \hat{\rho} = \hat{\kappa} / \hat{\sigma} )</td>
<td>8.95</td>
<td>8.27</td>
</tr>
<tr>
<td>( \hat{\alpha} \hat{L} / (\hat{\alpha} - \hat{\kappa} \hat{\lambda}) )</td>
<td>21.39</td>
<td>24.29</td>
</tr>
<tr>
<td>( \hat{\sigma}^2 + \hat{\kappa}^2 \hat{\lambda} )</td>
<td>0.003666</td>
<td>0.002169</td>
</tr>
</tbody>
</table>

#Observations 2771 2760

Asymptotic heteroskedasticity and autocorrelation consistent t-values in brackets; Newey and West (1994) with quadratic kernel.

\(^a\) Denotes significance at the 99% confidence level.

All our estimates of the five model parameters in Table 2 turn out to be significantly different from zero at the 99% confidence level. The results are evidence for a higher mean reversion rate for implied U.S. market volatility. Deviations from the respective mean reversion level are dampened by a force of nearly double strength in the VIX time series. A higher estimated reversion rate reveals that the influence of events causing increased volatility is less persistent than in the German market. As expected, variation in VIX is higher than variation in VDAX. This results in larger estimates of the standard deviation as well as the jump height. The total VIX error variance is roughly 1.7 times larger than the one of the VDAX. Note that while the jump-to-diffusion ratios are close to each other, the jump intensity for the VDAX is nearly double as high as the VIX intensity.

The estimated mean reversion levels for the two indices are roughly identical, yielding an estimate of 20.06% for the VIX and 20.54% for the VDAX. This result reflects that both underlying stock index portfolios are well diversified thereby having a small unsystematic risk component. However, calculating the long run expected volatility levels indicates that the 24.29% VDAX level well exceeds the level of 21.39% for the VIX. Hence, although the mean reversion levels—representing the basic risk level—are close to each other, a difference in volatility dynamics—namely a lower mean reversion rate combined with a higher jump frequency—enlarges the average level of the VDAX volatility index relative to the VIX.

6.3. Jump events

The analysis of jump events in the VIX and the VDAX time series is based on a synchronized number of 2697 observations. In case one of the indices is not calculated at a certain calendar day, a possible jump event in one market is not observable until the next following trading day at which both indices are quoted. Note further that—without any specific exchange closes—due to asynchronous daily trading times, a jump occurring at some date in
Table 3
Jump times and standardized jump sizes for VIX and VDAX, January 2, 1992 to December 30, 1997

<table>
<thead>
<tr>
<th>Date</th>
<th>VIX</th>
<th>VDAX</th>
<th>Event</th>
</tr>
</thead>
<tbody>
<tr>
<td>16 June 1992</td>
<td>3.44</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>17 July 1992</td>
<td>–</td>
<td>8.56</td>
<td>–</td>
</tr>
<tr>
<td>7 August 1992</td>
<td>–</td>
<td>7.58</td>
<td>–</td>
</tr>
<tr>
<td>21 August 1992</td>
<td>–</td>
<td>4.54</td>
<td>–</td>
</tr>
<tr>
<td>7 September 1992</td>
<td>–</td>
<td>3.83</td>
<td>–</td>
</tr>
<tr>
<td>15 September 1992</td>
<td>–</td>
<td>3.81</td>
<td>–</td>
</tr>
<tr>
<td>2 October 1992</td>
<td>–</td>
<td>3.41</td>
<td>–</td>
</tr>
<tr>
<td>21 October 1992</td>
<td>–</td>
<td>3.66</td>
<td>–</td>
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<tr>
<td>12 February 1993</td>
<td>4.26</td>
<td>–</td>
<td>–</td>
</tr>
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<td>13 September 1993</td>
<td>–</td>
<td>3.67</td>
<td>–</td>
</tr>
<tr>
<td>3 February 1994</td>
<td>7.57</td>
<td>–</td>
<td>Fed tightens monetary policy</td>
</tr>
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<td>28 March 1994</td>
<td>3.74</td>
<td>–</td>
<td>Worries: interest rates</td>
</tr>
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<td>23 June 1994</td>
<td>3.51</td>
<td>–</td>
<td>Worries: USD exchange rate</td>
</tr>
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<td>10 October 1994</td>
<td>–</td>
<td>3.62</td>
<td>Worries: expected automobile sales</td>
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<tr>
<td>15 December 1995</td>
<td>3.86</td>
<td>–</td>
<td>–</td>
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<td>–</td>
<td>Worries: USD exchange rate</td>
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<tr>
<td>8 March 1996</td>
<td>–</td>
<td>4.60</td>
<td>Worries: U.S. labor market report</td>
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<td>12 July 1996</td>
<td>4.18</td>
<td>–</td>
<td>Hewlett-Packard: profit warning</td>
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<tr>
<td>15 July 1996</td>
<td>–</td>
<td>4.50</td>
<td>Hewlett-Packard: profit warning</td>
</tr>
<tr>
<td>5 December 1996</td>
<td>–</td>
<td>5.61</td>
<td>Greenspan warning on rates</td>
</tr>
<tr>
<td>27 March 1997</td>
<td>–</td>
<td>6.20</td>
<td>Weak bond markets</td>
</tr>
<tr>
<td>28 May 1997</td>
<td>–</td>
<td>3.31</td>
<td>–</td>
</tr>
<tr>
<td>22 July 1997</td>
<td>–</td>
<td>4.20</td>
<td>Greenspan calms rates fear</td>
</tr>
<tr>
<td>22 October 1997</td>
<td>–</td>
<td>4.10</td>
<td>IFO business climate release</td>
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<td>24 October 1997</td>
<td>9.27</td>
<td>–</td>
<td>Asian currency crisis</td>
</tr>
<tr>
<td>27 October 1997</td>
<td>–</td>
<td>6.79</td>
<td>Asian currency crisis</td>
</tr>
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</table>

# Jumps 10 (7′) 17 (12′)

All jumps are recorded at the 0.05% significance level. (′) Denotes jump observation at the higher 0.01% significance level. (*) Denotes synchronous jump event occurring first for the VIX. (**) Denotes synchronous jump event occurring first for the VDAX.

the VIX may cause a jump in the VDAX on the following trading day, whereas a causality in the opposite direction is characterized by a jump event at the same trading day.

6.3.1. Jump event detection

Given the simplifying assumption of known parameters $\theta$ of our volatility model, application of the results of Section 5 allows us to detect jump events in our given sample. For the given individual VIX and VDAX samples, the parameter estimates $\hat{\sigma}^2$, $\hat{k}$ and $\hat{\lambda}$ are used to derive the standardized residuals $\hat{\xi}_t$. Choosing significance levels of $e = 0.05\%$ and $e = 0.01\%$, the corresponding empirical jump thresholds $b$ are calculated for each series.

Tables 3 and 4 summarize the resulting jump dates and the standardized jump event residuals. We report 20 jump events for the VIX and 31 jump events for the VDAX at the 0.05% significance level. Increasing the significance level to 0.01% results in 14 big jump events for the VIX and 20 big jump events for the VDAX. Irrespective of the significance
Table 4
Jump times and standardized jump sizes for VIX and VDAX, January 2, 1998 to December 30, 2002

<table>
<thead>
<tr>
<th>Date</th>
<th>VIX</th>
<th>VDAX</th>
<th>Event</th>
</tr>
</thead>
<tbody>
<tr>
<td>8 January 1998</td>
<td>5.57</td>
<td>–</td>
<td>Asian currency crisis aftershock</td>
</tr>
<tr>
<td>26 May 1998</td>
<td>–</td>
<td>4.26</td>
<td>Corporate earnings releases</td>
</tr>
<tr>
<td>12 June 1998</td>
<td>–</td>
<td>3.28</td>
<td>Corporate earnings releases</td>
</tr>
<tr>
<td>22 July 1998</td>
<td>3.78</td>
<td>–</td>
<td>Greenspan talk on the economy</td>
</tr>
<tr>
<td>10 August 1998</td>
<td>–</td>
<td>3.55</td>
<td>–</td>
</tr>
<tr>
<td>21 August 1998</td>
<td>–</td>
<td>7.25</td>
<td>Russian debt crisis</td>
</tr>
<tr>
<td>26 August 1998</td>
<td>4.93</td>
<td>–</td>
<td>Russian debt crisis</td>
</tr>
<tr>
<td>9 September 1998</td>
<td>3.89</td>
<td>–</td>
<td>Worries: President Clinton</td>
</tr>
<tr>
<td>16 September 1998</td>
<td>–</td>
<td>3.29</td>
<td>SAP: profit warning</td>
</tr>
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<td>1 October 1998</td>
<td>–</td>
<td>4.23</td>
<td>Worries: USD exchange rate</td>
</tr>
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<td>30 November 1998</td>
<td>–</td>
<td>5.36</td>
<td>Worries: USD exchange rate</td>
</tr>
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<td>12 January 1999</td>
<td>–</td>
<td>3.93</td>
<td>–</td>
</tr>
<tr>
<td>21 May 1999</td>
<td>–</td>
<td>3.29</td>
<td>–</td>
</tr>
<tr>
<td>3 January 2000</td>
<td>3.55</td>
<td>–</td>
<td>Fed unexpectedly lowers rates</td>
</tr>
<tr>
<td>5 July 2001</td>
<td>–</td>
<td>3.32</td>
<td>German order books report</td>
</tr>
<tr>
<td>11 September 2001</td>
<td>–</td>
<td>7.35</td>
<td>September 11 terror attacks</td>
</tr>
<tr>
<td>13 September 2001</td>
<td>–</td>
<td>4.86</td>
<td>September 11 terror attacks</td>
</tr>
<tr>
<td>17 September 2001</td>
<td>5.82</td>
<td>–</td>
<td>September 11 terror attacks</td>
</tr>
<tr>
<td>30 August 2002</td>
<td>4.04</td>
<td>–</td>
<td>NAPM Index release: Chicago</td>
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<tr>
<td>3 September 2002</td>
<td>–</td>
<td>3.87</td>
<td>NAPM Index release: national</td>
</tr>
<tr>
<td>27 September 2002</td>
<td>–</td>
<td>3.61</td>
<td>–</td>
</tr>
</tbody>
</table>

# Jumps 10 (7) 14 (8)

All jumps are recorded at the 0.05% significance level. (’) Denotes jump observation at the higher 0.01% significance level. (**) Denotes synchronous jump event occurring first for the VIX. (***) Denotes synchronous jump event occurring first for the VDAX.

level, there are four synchronous jump events occurring during the 11 years of our sample. Hence, the ratio of spillover (i.e. joint) to local (i.e. specific) jump events is 4/20 for the VIX and 4/31 for the VDAX. This indicates that the filtered jumps in ex ante risk expectations were caused by country specific events much more often than by global market events. A graphical representation of the jumps is given in Fig. 2. The jump dates indicate clustering of volatility shocks for both markets, i.e. there is a tendency of one shock being followed by another. The figure also illustrates that most shocks are local with only relatively few shocks occurring at identical or very close dates.

6.3.2. Jump event public news

Given the jump dates of Tables 3 and 4, we use the Factiva database (http://www.factiva.com) in order to screen for related public news. The search results indicate the type of news event which occurred on the particular jump days. The results are quoted in the last column of Tables 3 and 4.

The four market spillover shocks include the events of the Asian currency crisis in fall 1997 as well as the September 11, 2001 terror attacks in the U.S. The two other spillover
shocks are given under U.S. market related news events, namely a U.S. labor market report in March 1996 and a Hewlett-Packard profit warning in July 1996. Interestingly, all spillover shocks with exception of September 11 first occur for the VIX and then for the VDAX. All other shocks reported represent local events which in part can be assigned particular public news. In case this is possible, the news can be categorized to include four groups of news events. Most importantly this includes worries about as well as actual unexpected releases concerning U.S. monetary policy and interest rates (eight reported events, thereof seven releases, one report on bond market weakness). Next are releases on macroeconomic data and corporate profits including single profit warnings which all provide potential cause for volatility shocks in our sample (five reported events, respectively). Whereas the former events all but one relate to news releases (one bond market exception), the reported worries about prevailing exchange rate relations (five reported events) document the impact of external market movements. Here, our results suggest that the bond and particularly the foreign exchange markets play a predominant role in explaining shocks to equity market risk.

7. Conclusion

The present study proposes a straightforward method for detecting jumps in implied volatility. Since such jumps are crucial in many applications of financial theory, it seems worthwhile to test for jump events in implied volatility as well as to consider their relation to public news events.

The finding of a significant and positive jump component in implied market volatility has several implications. The following areas may offer an illustration of potential tasks for further investigation. First, asset pricing theory predicts that changes in market risk measured by volatility should affect expected asset returns which may also hold for jumps in
risk expectations. The relationship between changes in the level of volatility and unexpected stock returns was examined in an empirical study, e.g. by Haugen et al. (1991). Second, risk management—and volatility forecasting in particular—should account for asymmetric error distributions stemming from positive jumps in volatility. Third, jumps in implied market volatility have an impact on stock as well as on volatility option pricing. Eraker et al. (2003) document that—once jumps in prices and volatility are considered—the pricing implications for options on assets may coincide well with the typically observed smiles in implied volatility.

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References