Extreme Asymmetric Volatility, Leverage, Feedback and Asset Prices

Sofiane Aboura
Université de Paris, Dauphine

—

Niklas Wagner
Passau University

3rd Risk Management Conference
Singapore, July 17, 2009

Motivation I

"Members of the Federal Reserve's policy-setting committee worried at their most recent meeting that housing and financial market stress could trigger a nasty slide in the economy."

Reuters, April 8, 2008

Motivation II

- Asymmetric volatility in equity markets
- Explain extreme market movements
- Model of
  - market returns
  - conditional market volatility
  - volatility of volatility (“VoV”)
- Test for extreme asymmetry
- Derive asset pricing implications

Asymmetric Volatility I

- Three features:
  - (i)*negative* relation between realized returns and conditional volatility
  - (ii)*positive* relation between conditional expected market returns and volatility
  - (iii)*asymmetry* in that (i) is more pronounced for negative returns

Asymmetric Volatility II

- Common explanations:
  - financial *leverage* hypothesis (e.g. Black (1976) and Christie (1982))
  - volatility *feedback* hypothesis (e.g. Black (1976), Pindyck (1984), French, Schwert, and Stambaugh (1987), Campbell and Hentschel (1992) and Beakert and Wu (2000))
- Feedback:
  - given a large piece of unexpected bad news, volatility increases and feedback yields an additional decrease in prices, which in turn amplifies the initial shock,
  - i.e.: extreme feedback represents systemic risk

Model I

The model of returns is

$$R_t = \mu + Y_t \sigma_t. \tag{2.1}$$

For volatility we assume

$$\Delta \ln \sigma_t = \sum_{j=1}^{p} \delta_j \Delta \ln \sigma_{t-j} + f(Y_{t-1}; \sigma_{t-1}) + Z_t \phi_t.$$

Innovations (Y_t, Z_t) drive the return-volatility model.

Model II

Rearrange the above equation and obtain for $\ln \sigma_t$:

$$\ln \sigma_t = (1+\delta_1) \ln \sigma_{t-1} - \delta_1 \ln \sigma_{t-2} + \sum_{j=2}^{p} \delta_j \Delta \ln \sigma_{t-j}$$

$$+ f(Y_{t-1}; \sigma_{t-1}) + Z_t \phi_t. \tag{2.2}$$

Specify parametric functions f in equation (2.2), motivated by threshold ARCH models, $\theta \neq 0$, choose:

$$\begin{aligned}
f_1 &= \omega_0 + \omega_1 |Y_{t-1}\sigma_{t-1}| + \omega_2 |Y_{t-1}\sigma_{t-1}| I_{\{Y_{t-1}\sigma_{t-1}<0\}}, \\
f_2 &= \omega_0 + \omega_1 |Y_{t-1}| + \omega_2 |Y_{t-1}| I_{\{Y_{t-1}<0\}}, \\
f_3 &= \omega_0 + \omega_1 |Y_{t-1}\sigma_{t-1}| + \omega_2 |Y_{t-1}\sigma_{t-1}| I_{\{Y_{t-1}\sigma_{t-1}<\theta\}}.
\end{aligned}$$

Model III

Conditional variance of the logarithmic conditional market volatility, ϕ_t^2, is assumed to follow an asymmetric GJR-GARCH equation:

$$\phi_t^2 = \gamma_0 + \gamma_1 (Z_{t-1}\phi_{t-1})^2 +$$

$$\gamma_2 (Z_{t-1}\phi_{t-1})^2 I_{\{Z_{t-1}\phi_{t-1}<0\}} + \gamma_3 \phi_{t-1}^2. \quad (2.3)$$

Return-volatility model (2.1-3).

Dependence I

Asymmetric volatility

- time-varying nature of the correlation between realized returns and conditional volatility
- extreme leverage and feedback

Model of dependence in the innovations (Y_t, Z_t)

- multivariate GARCH: DCC Engle (2002) and Engle and Sheppard (2001)
- extreme value theory (EVT): bivariate GPD, e.g. Coles (2001)

Dependence II

Asymptotic dependence versus independence:

Consider the limiting conditional probability

$$\chi = \lim_{u \to \infty} P(Y > u | Z > u).$$

Two cases:

- $\chi = 0$: asymptotic independence,
- $\chi > 0$: asymptotic dependence.

Dependence III

Coles, Heffernan and Tawn (1999):

$\chi = 0$: $\quad \chi(u) \to 0$ and $\overline{\chi}(u) \to const. < 1$

$\chi > 0$: $\quad \chi(u) \to const. > 0$ and $\overline{\chi}(u) \to 1$

Falk and Michel (2006):

$$\lim_{u \to \infty} P(Y + Z \leq t | Y + Z > u) = \begin{cases} t^2 : \text{iff} \quad \chi = 0 \\ t : \text{otherwise} \end{cases}$$

Data Set

- S&P 500 closing prices yield a sample of R_t
- VIX market volatility index yields a sample of σ_t
- Period: January 2, 1990 to September 30, 2008
- $T = 4890$ daily observations
- Stress periods: 1997, 1998, 2000 to 2002 and 2008
- Model (2.1-3) yields sample of (Y_t, Z_t)
- Joint extremes of interest: $(Y, -Z)$ and $(-Y, Z)$

Empirical Results I

Model

- VIX is observable $\Rightarrow Y_t$
- VoV: MLE parameter estimates $\Rightarrow Z_t$
- Model time series, R_t, σ_t, ϕ_t
- $\longrightarrow$ Figure 1
- Residual diagnostics for (Y_t, Z_t)
- $\longrightarrow$ iid assumption reasonable

Empirical Results II

Asymmetric volatility hypotheses

- $\mathrm{H}_0^{(1)}: \quad \bar{\rho} \geq 0$
- $\mathrm{H}_0^{(2)}: \quad (Y_t, -Z_t)$ asy. ind.
- $\mathrm{H}_0^{(3)}: \quad (-Y_t, Z_t)$ asy. ind.
- $\mathrm{H}_0^{(4)}: \quad (-Y_{t-1}, Z_t)$ asy. ind.
- $\mathrm{H}_0^{(5)}: \quad (-Y_t, Z_{t-1})$ asy. ind.
- $\mathrm{H}_0^{(6)}: \quad (-Y_t, Z_t)$ sym. dep. rel.
- $\mathrm{H}_0^{(7)}: \quad \lim_{u \to \infty} P(-Y_t > u | Z_t > u) = 0$

Empirical Results III

Results

- $\mathrm{H}_0^{(1)}$: $\overline{\rho} \geq 0$: *rej.: asy. vol.*
- $\mathrm{H}_0^{(2)}$: $(Y_t, -Z_t)$ asy. ind.: not rej.
- $\mathrm{H}_0^{(3)}$: $(-Y_t, Z_t)$ asy. ind.: *rej.: ex. asy. vol.*
- $\mathrm{H}_0^{(4)}$: $(-Y_{t-1}, Z_t)$ asy. ind.: not rej.
- $\mathrm{H}_0^{(5)}$: $(-Y_t, Z_{t-1})$ asy. ind.: not rej.
- $\mathrm{H}_0^{(6)}$: $(-Y_t, Z_t)$ sym. dep. rel.: not rej.
- $\mathrm{H}_0^{(7)}$: $\lim_{u \to \infty} P(-Y_t > u | Z_t > u) = 0$: *rej.: feedb.*

Empirical Results IV

GPD results

- bivarite GPD model for $(-Y_t, Z_t)$
- joint censored MLE's
- $\rightarrow$ high quantile exceedance probabilities

Asset Pricing Implications I

Extreme volatility feedback probability:

- Given large unexpected shock, $Z_t > z$
- extreme volatility feedback: $P(-Y_t > -y | Z_t > z) = s$
- expected negative market return $\mathbb{E}(-Y_t | -Y_t > -y)$
- no feedback: $1 - s$
- $\rightarrow$ Table 6

Asset Pricing Implications II

Extreme volatility feedback impact:

- volatility shock, $Z_t > z$
- return-volatility model (2.1-2)
- set volatility and VoV (2.2) equal to unconditional means
- e.g. start with: $\overline{\sigma} = 0.0121$, i.e. 19.1 percent annually
- then vary market volatility
- $\rightarrow$ Figure 6

Conclusion

- New model: market returns and observable conditional volatility
- Test implications of leverage and feedback: large unexpected return and volatility shocks
- Extreme asymmetric volatility: documented component of systemic risk
- Large market declines: partly explainable via volatility feedback
- Stabilization of conditional market volatility: task for market regulators