Stress Testing and Risk Integration in Banks

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Scope of the Book

The book pursues the following objectives:

1. Describe **how to conduct** a stress testing exercise.

2. Introduce alternative **risk integration frameworks**.

3. Highlight the key steps to perform a **reverse stress test**.

- It is aimed at graduates, master students and practitioners.

- It also provides hints for **further research** and applications in statistics and finance.

Toolkit

When dealing with risk management (and stress testing in particular), one may be dragged into the details of a specific topic. **The risk is to lose the broader blueprint**. The book presents a global view of a bank by relying on the following toolkit:

1. **Formal** mathematical representation.

2. Use of a **bank prototype** throughout the entire stress testing and risk integration journey.

3. **Business cases** investigation.

4. **Worked examples** (and worked exercises) in Matlab and R.

Framework Outline

The following key areas are covered to achieve book's goals:

1. Introduce multivariate **time series analysis** to highlight how to enrich a given scenario and generate Monte Carlo simulations.

2. Describe the key features of **asset and liability management** to assess interest rate and liquidity risks.

3. Investigate **Portfolio models**.

4. Assess stress testing **impacts** (e.g., capital ratios, risk weighted assets, etc.).

5. Describe potential **limitations of the current stress testing process**.

Agenda

- ▶ Introduction

- ▶ Macroeconomic Transmission Mechanics

- ▶ Stress Testing

- ▶ Risk Integration and Reverse Stress Testing

- ▶ Concluding Remarks

Agenda

- **Introduction**

    - What Went Wrong

        - Lehman Brothers business case
        - Northern Rock business case

    - Regulatory Responses
        - Capital, leverage and liquidity ratios

    - Book Structure at a Glance
        - Functional perspective
        - Risk metrics

Lehman Brothers - Business Case

▶ In the 2000s, Lehman Brothers widely expanded its services.

▶ During the period 2006-2007, Lehman Brothers initiated a new strategy. **The company aggressively bought real estate assets**.

▶ At the end of its 2007 fiscal year, Lehman Brothers held $111 billion in commercial or residential assets and securities. More than double the $52 billion that it held at the end of 2006, and more than four times its equity.

▶ Market illiquidity and massive losses in this business caused rating agencies and investor to express concerns in 2008.

▶ During that summer, **Lehman Brothers financial situation become unsustainable**. The crisis became public.

Black Rock - Business Case

▶ Northern Rock was a building society with challenging ambitions.

▶ In the nine years after its going public to the eve of the crisis in June 2007, its total assets grew from £ 17.4 billion to £ 113.5 billion.

▶ Its liability structure reflected this unusual dynamic by heavily depending on non-retail funds.

▶ By the summer of 2007, only 23 percent of its liabilities were in the form of retail deposits.The rest of its funding came from short-term borrowing in the capital markets, or through **securitised notes (i.e., asset backed commercial paper -ABCP-)**.

▶ In the summer 2007, the sub-prime crisis started affecting banks' balance sheets. **The demize of Northern Rock dated from August 9**.

Capital Strengthening and Leverage

► Strengthening of risk-based capital ratios as a buffer against unexpected losses (e.g., CET1 ratio $\geq$ 4.50 %)

$$Capital \quad Ratio = \frac{Regulatory \quad Capital}{RWA}. \tag{1}$$

► Introduction of the leverage ratio as additional requirement

$$Leverage \quad Ratio = \frac{Capital \quad Exposure}{Exposure \quad Measure} \geq 3.00\%. \tag{2}$$

Hints on RWA | **Standardized Approach - Credit**

- ▶ The concept of RWA is crucial throughout the book. In what follows a rough idea is provided to familiarize with this concept.
- ▶ Let us consider a $ 10 billion credit portfolio spread among the regulatory categories detailed in the next table. For each group of **exposures ($\sum_{i \in ss} EXP_{i,ss}$)** a **specific risk weight** W_{ss} is required to compute the corresponding RWA. The last two columns summarize how to perform the calculation.

Assets	$\sum_{i \in ss} EXP_{i,ss}$	W_{ss}	RWA
Claims on corporations	4.00	100.00 %	4.00
Retail exposures	2.00	75.00 %	1.50
Residential real estate	1.00	35.00 %	0.35
Sovereign and central banks	3.00	20.00 %	0.60
Total	10.00		6.45

Table: Standardized credit RWA computation ($ billions).

Liquidity

- Introduction of liquidity coverage ratio

$$LCR = \frac{High \quad Quality \quad Liquid \quad Assets}{Total \quad Net \quad Cash \quad Outflows} \geq 100.00\%. \quad (3)$$

- Introduction of net stable funding ratio

$$NSFR = \frac{Available \quad Stable \quad Funding}{Total \quad Net \quad Cash \quad Outflows} \geq 100.00\%[1]. \quad (4)$$

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[1]Under consultation until 2018.

Stress Testing Process

▶ In the US the **Comprehensive Capital Analysis and Review (CCAR)**, was introduced as a key component of the Dodd-Frank Act (2010).

▶ In Europe, the **European Central Bank** (together with the European Banking Authority, EBA) deployed a first wide-stress testing exercise in 2011 which was then replicated in 2014 and 2016.

▶ The **Bank of England**, in 2014 aligned with the Federal Reserve Board committing an annual stress testing process for major UK banks.

Book Structure

1. A key step in the stress testing journey is to highlight the **connection between the overall economy and a given bank**.

2. A bank can be explored by means of its asset and liability structure with focus on: **trading book, banking book, liabilities and own funds**.

3. Each of these areas affects **capital, leverage and liquidity ratios**. They also are the key ingredients of **risk integration and reverse stress testing processes**.
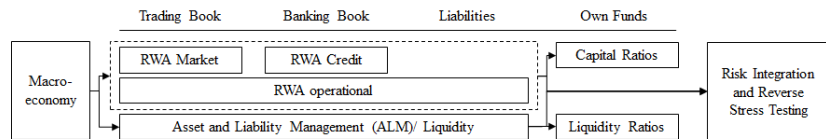
Figure: Book structure from a functional perspective.

Chapters' Organisation

▶ A matrix scheme has been used by considering **asset and liability** together with key **risk management metrics** as journey's cornerstones.
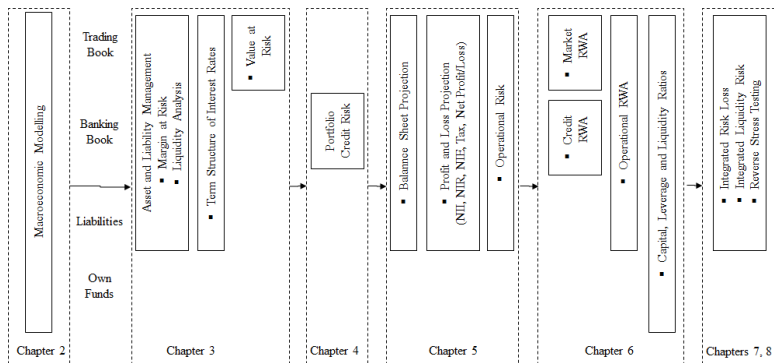
Figure: Intersection between risk metrics and bank functional organization.

Agenda

- **Macroeconomic Transmission Mechanics**

    - Macroeconomic Modelling
        - VAR and GVAR

    - Bank Alpha
        - Regulatory Capital
        - Risk Weighted Assets (RWAs)
        - Leverage and Liquidity

VAR and GVAR Modelling

▶ The VAR is introduced after discussing the univariate analysis (AR, MA and ARMA models). Let $\mathbf{x}_t = (x_{1,t}, \ldots, x_{k,t})'$ denote a $p \times 1$ vector of macroeconomic time series

$$\mathbf{x}_t = \mathbf{c} + \boldsymbol{\Phi}_1 \mathbf{x}_{t-1} + \ldots + \boldsymbol{\Phi}_k \mathbf{x}_{t-k} + \boldsymbol{\epsilon}_t, \tag{5}$$

▶ In a global economy, a GVAR helps capturing international linkages. The following equations are representative of a GVAR(2,2)

$$\begin{aligned} \mathbf{x}_{s,t} = \mathbf{c}_{s,0} + \mathbf{c}_{s,1}t + \boldsymbol{\Phi}_{s,1}\mathbf{x}_{s,t-1} + \boldsymbol{\Phi}_{s,2}\mathbf{x}_{s,t-2} + \\ + \boldsymbol{\Lambda}_{s,0}\mathbf{x}_{s,t}^* + \boldsymbol{\Lambda}_{s,1}\mathbf{x}_{s,t-1}^* + \boldsymbol{\Lambda}_{s,2}\mathbf{x}_{s,t-2}^* + \mathbf{u}_{s,t}, \end{aligned} \tag{6}$$

where $\mathbf{x}_{s,t}$ is the $p_s \times 1$ vector of country specific variables, $\mathbf{x}_{s,t}^*$ is the $p_s^* \times 1$ vector of foreign variables. $\mathbf{c}_{s,0}$ is the constant term, while $\mathbf{c}_{s,1}$ is the trend coefficient. $\boldsymbol{\Phi}_{s,(\cdot)}$ and $\boldsymbol{\Lambda}_{s,(\cdot)}$ are the matrix coefficients for the domestic and foreign variables. $\mathbf{u}_{s,t}$ represents the error component. Matlab **GVAR Toolbox** is used for estimates.

Conditional Forecast

▶ The conditional VAR can intuitively be represented as follows in a VAR(1) framework

$$\mathbf{x}_t = \mathbf{c} + \mathbf{\Phi}_1 \mathbf{x}_{t-1} + \epsilon_t, \tag{7}$$

$$\mathbf{x}_{t+1} = \mathbf{c} + \mathbf{\Phi}_1 \mathbf{x}_t + \epsilon_{t+1}, \tag{8}$$

therefore

$$\mathbf{x}_{t+1} = \mathbf{c} + \mathbf{\Phi}_1 [\mathbf{c} + \mathbf{\Phi}_1 \mathbf{x}_{t-1} + \epsilon_t] + \epsilon_{t+1}, \tag{9}$$

and more generally

$$\mathbf{x}_{t+n} = \mathbf{c} \sum_{j=0}^{n-1} \mathbf{\Phi}_1^j + \mathbf{\Phi}_1^n \mathbf{x}_t + \sum_{j=1}^{n} \mathbf{\Phi}_1^{n-j} \epsilon_{t+j}. \tag{10}$$

Bank Alpha at a Glance

▶ **Bank Alpha is bank prototype** used throughout the book to nail down the key concepts explored along the journey.

▶ The following balance sheet summarizes its key accounting measures in t_0 (i.e., stress testing starting point).

Assets		Liabilities	
Cash resources	8.00	Deposits	70.00
Securities	14.00	Others	17.00
Loans	70.00	Subordinated debts	4.00
Others	8.00	Non controlling interests	2.00
		Shareholder equity	7.00
Total assets	100.00	Total liabilities	100.00

Table: Bank Alpha's balance sheet in t_0 ($ billions).

Metrics to be Stressed | **Capital**

▶ Regulatory capital is computed by applying adjustments starting from accounting **shareholder equity**.

▶ The following table summarizes Bank Alpha's regulatory capital in t_0.

Regulatory Capital t_0	
Common equity tier 1 (CET1)	4.44
Additional Tier 1	1.00
Tier 1	5.44
Tier 2	6.00
Total capital	11.44

Table: Bank Alpha's regulatory capital in t_0 (\$ billions).

Metrics to be Stressed | **RWA**

▶ Capital ratios' denominator (i.e., RWA) is the **sum of market, credit and operational risk** weighted assets.

RWA	t_0
Market risk	2.31
Operational risk	8.93
Credit risk	54.80
Other risks	1.20
Shortfall	2.50
Add-on RWA	6.26
Total RWA	76.00

Table: Bank Alpha's RWA in t_0 (\$ billions).

▶ In t_0, Bank Alpha exceeds the 4.50 % regulatory threshold as shown below

$$Bank \quad Alpha \quad CET1 \quad Ratio = \frac{4.44}{76.00} = 5.84\%$$

Metrics to be Stressed | **Leverage and Liquidity**

▶ From a structural point of view, in t_0 Bank Alpha **exceeds the minimum 3 % leverage ratio** (i.e., ratio between capital exposure and total asset exposure).

▶ It also exceeds the minimum **liquidity coverage ratio (LCR)** minimum threshold 100 %. The net stable funding ratio is below the 100 % target.

	t_0
Leverage ratio	5.53 %
LCR	104.00 %
NSFR	88.25 %

Table: Bank Alpha's leverage and liquidity ratios.

Macroeconomic Scenario for Bank Alpha

▶ Scenario key features: increase of US unemployment rate, low interest rates, sharp reduction of GDP followed by rapid reinforcement, J-shape inflation path.
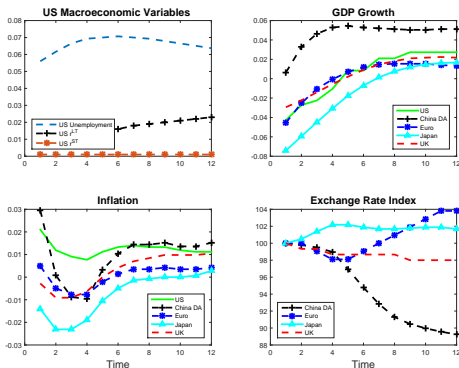
Figure: Macroeconomic scenario.

Agenda

- **Stress Testing**

    - RWA Stress Testing
        - Market Risk RWA
        - Credit Risk RWA
        - Operational Risk RWA

    - Balance Sheet Projection
        - Credit Life Cycle
        - Balance Sheet Projection vs. Macro Scenarios
        - Profit and Loss Projection

    - Bank Alpha's Key Results

Market Risk RWA | **Normal Return Distribution**

▶ Let us consider an equity with a daily standard deviation of $ 10 million. The 99 % confidence interval VaR is $ 23 million when using the assumption of normal distribution of returns.

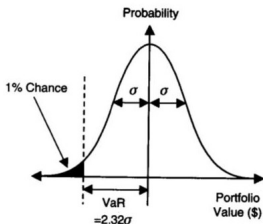▶ One would expect that losses would be greater than $ 23 million on 1 % of trading days (i.e., 2 or 3 days per year).

Figure: General idea behind VaR.

Market Risk RWA | **Value at Risk**

▶ The VaR at level $(1 - \alpha)$, is computed as the smallest loss (ℓ) such that

$$VaR_{(1-\alpha)} = \inf_{\ell} P(Loss_{\Delta R} > \ell) \leq \alpha. \qquad (11)$$

▶ The variance-covariance approach is usually adopted in portfolios with linear pay-offs where returns have variance $\boldsymbol{w'\Sigma w}$ ($\boldsymbol{w}$ is the vector of weights and $\boldsymbol{\Sigma}$ is the variance covariance matrix of returns).

▶ The following equation summarizes VaR by relying on the usual assumption to multiply the 1-day VaR times the square root of t

$$VaR_{(1-\alpha),t} = \Phi^{-1}(\alpha)\sqrt{\boldsymbol{w'\Sigma w}}\sqrt{t}. \qquad (12)$$

Market Risk RWA | **Stressed VaR**

▶ When relying on the variance covariance approach, extreme market conditions are taken into account by replacing the current variance covariance matrix with the stressed one.

▶ Let us consider the vector of returns **r**, the current covariance matrix **Σ** and the covariance under stress **W**.

▶ Stressed returns are simulated by relying on the Cholesky decomposition of **Σ** and **W** denoted respectively **C** and **D** as follows

$$\mathbf{r}_{stress} = \mathbf{C}\mathbf{D}^{-1}\mathbf{r}. \tag{13}$$

▶ The term structure of interest rate is also required to stress asset and liability elements. Therefore models such as, for example, CIR are fitted to historical data and, then, projected by considering **macroeconomic scenario constraints**.

Market Risk RWA | **Regulatory Formula**

▶ For regulatory purposes, the following equation holds

$$VaR_{reg,t} = \max \left(VaR_{(1-\alpha),t}, \frac{1}{60} \sum_{r=1}^{60} VaR_{(1-\alpha),t-r} \cdot MM \right), \quad (14)$$

where, $VaR_{(1-\alpha),t}$ is the end period 10-day VaR. The other item within the *max* formula (i.e., $\frac{1}{60} \sum_{r=1}^{60} VaR_{(1-\alpha),t-r} \cdot MM$) is the average 10-day VaR measured over the 60 days before the end of period. *MM* is a multiplier ranging between 3 and 4, based upon the number of back-testing exceptions that occur on a rolling twelve month period.

▶ The (advanced method) RWA is computed as follows

$$RWA_{mkt} = 12.5(VaR_{reg} + SVaR_{reg} + IRC + SSRC), \quad (15)$$

where **IRC** stands for incremental risk charge and is aimed to cover default and credit migration risks, while **SSRC** stands for standard specific risk charge and is substantially a residual risk.

Credit Risk RWA | **Portfolio Modelling - Sketch**

▶ The loss distribution for a portfolio of credits is built by relying on the principle that the $i - th$ customer of a portfolio may be insolvent and, in the case of default, its exposure may not be fully recovered.
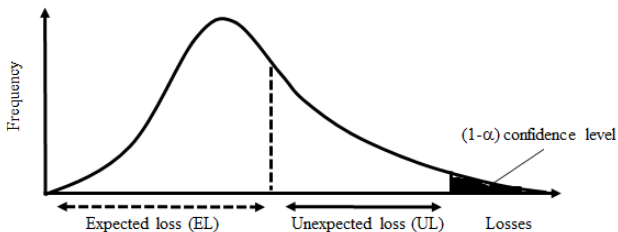
Figure: Loss distribution. Expected loss (EL) vs. unexpected loss (UL).

Credit Risk RWA | **Portfolio Modelling - Framework**

▶ The probability of default (PD_i) represents the likelihood of the insolvency. The loss given default (LGD_i) accounts for the non-recovered portion of an asset exposure (A_i).

▶ If we generate defaults through a Monte Carlo process, for each simulation $g = 1, \ldots, G$, the portfolio credit loss ($CLoss_g$) is computed as follows

$$CLoss_g = \sum_{i=1}^{n} \mathbb{1}_{i,def,g} A_i, LGD_i. \tag{16}$$

▶ **Unconditional default probability**. The unconditional one-year (1_y) PD for a bank's customer is defined as follows

$$PD_{i,s,t} = P(\tau_{i,s} \leq 1_y), \tag{17}$$

where $\tau_{i,s}$ denotes the default time occurrence.

Credit Risk RWA | **Advanced Formula (AIRB)**

▶ A binary default variable is the starting point for IRB modelling

$$\mathbb{1}_{i,s,def}^{IRB} = \left\{ \begin{array}{lll} 1 & for & \xi_{i,s,t} \leq \Phi^{-1}(PD_{i,s,t}), \\ 0 & for & \xi_{i,s,t} > \Phi^{-1}(PD_{i,s,t}), \end{array} \right. \tag{18}$$

where Φ is the normal cdf. Then, for the scenario x_{scen}

$$
\begin{aligned}
P\left(\mathbb{1}_{i,s,def}^{IRB} = 1 | \varkappa_{scen}\right) &= P\left(\xi_{i,s,t} \leq \Phi^{-1}(PD_{i,s,t}) | \varkappa_{scen}\right) = \\
&= P\left(\sqrt{\rho_{i,s}}\varkappa_t + \sqrt{1-\rho_{i,s}}\epsilon_{i,s,t} \leq \Phi^{-1}(PD_{i,s,t}) | \varkappa_{scen}\right) = \\
&= P\left(\epsilon_{i,s,t} \leq \frac{\Phi^{-1}(PD_{i,s,t}) - \sqrt{\rho_{i,s}}\varkappa_t}{\sqrt{1-\rho_{i,s}}} | \varkappa_{scen}\right) = \\
&= \Phi\left(\frac{\Phi^{-1}(PD_{i,s,t}) - \sqrt{\rho_{i,s}}\varkappa_t}{\sqrt{1-\rho_{i,s}}} | \varkappa_{scen}\right) = \Phi\left(\frac{\Phi^{-1}(PD_{i,s,t}) - \sqrt{\rho_{i,s}}\varkappa_{scen}}{\sqrt{1-\rho_{i,s}}}\right).
\end{aligned}
\tag{19}
$$

▶ The advanced IRB credit risk RWA is as follows

$$RWA \approx 12.5 \sum_{i=1}^{n} A_{i,s,t} LGD_{i,s,t} \Phi\left(\frac{\Phi^{-1}(PD_{i,s,t}) + \sqrt{\rho}\Phi^{-1}(0.999)}{\sqrt{1-\rho}} - PD_{i,s,t}\right). \tag{20}$$

Credit Risk RWA | **How to Stress PDs**

▶ The probability that a debtor i belonging to sector s will default within a certain period (e.g., one year) may be written as follows in a logit framework

$$PD_{i,s,t} = \frac{1}{1 + e^{-\Psi_{i,s,t}}}. \tag{21}$$

▶ The following indicator function summarizes the model structure

$$\mathbb{1}_{i,s,t,def}^{Logit} = \begin{cases} 1 & for & \frac{1}{1+e^{-\Psi_{i,s,t}}} \geq u_{i,s,t}, \\ 0 & for & \frac{1}{1+e^{-\Psi_{i,s,t}}} < u_{i,s,t}, \end{cases} \tag{22}$$

where $u_{i,s,t} \in [0,1]$ is a uniform random variable (s=sector).

▶ More precisely, the relationship between the creditworthiness index $\Psi_{i,s,t}$ and the vector of macro variables, may be written as follows

$$\Psi_{i,s,t} = \beta_{0,s} + \beta_{1,i,s} x_{1,t} + \ldots + \beta_{p,i,s} x_{p,t} + \sigma_{i,s} \epsilon_{i,s,t}. \tag{23}$$

▶ A macro variable shock is used to affect PDs through equation (21)

Credit Life Cycle | **Big Picture**

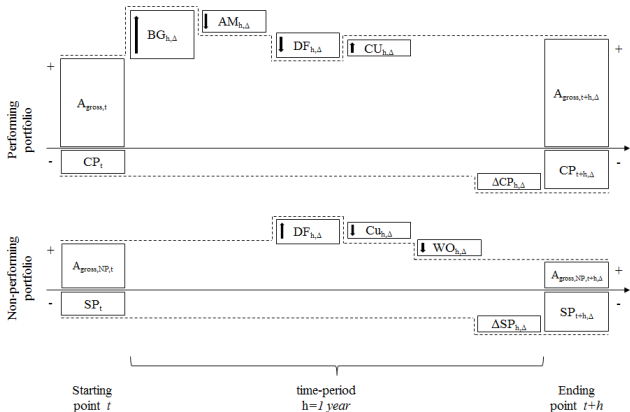▶ The typical life cycle for a typical commercial bank is represented as follows.

Figure: Credit life cycle dynamics over a one-year time horizon.

Credit Life Cycle | **Overall Framework**

▶ The performing portfolio dynamic can be summarized as follows

$$A_{gross,t+h,\Delta} = A_{gross,t} + BG_{h,\Delta} - AM_{h,\Delta} - DF_{h,\Delta} + CU_{h,\Delta}. \quad (24)$$

▶ The collective provisions stock evolution may be summarize d as described below

$$CP_{t+h,\Delta} = CP_t + \Delta CP_{h,\Delta}. \quad (25)$$

▶ Moving to the defaulted portfolio, the following holds

$$A_{gross,NP,t+h,\Delta} = A_{gross,NP,t} + DF_{h,\Delta} - CU_{h,\Delta} - WO_{h,\Delta}. \quad (26)$$

▶ Finally, specific provisions dynamic can be represented in the following way

$$SP_{t+h,\Delta} = SP_t + \Delta SP_{h,\Delta}. \quad (27)$$

Balance Sheet Projection vs. Macro Scenarios

▶ Exposure $A_{k,t}$ and macroeconomic variables are linked as follows

$$A_{k,t} = \beta_{k,0} + \beta_{k,1}x_t + \ldots + \beta_{k,p}x_{p,t} + \epsilon_{k,t}, \tag{28}$$

whereby $\boldsymbol{\beta}_k = (\beta_{k,1}, \ldots, \beta_{k,p})'$ is the vector of coefficients fitted against the vector of macroeconomic variables $\mathbf{x}_t = (x_{1,t}, \ldots, x_{p,t})'$. Other credit life cycle components are derived to get the overall balance sheet evolution.

▶ The trading book may be projected as follows

$$\begin{cases} TB_{t+h} = TB_t + \Delta TB_h, \\ \Delta TB_h = f(RA_t, \mathbf{x}_{h,\Delta}), \end{cases} \tag{29}$$

where TB_t is the trading book volume at time t, while ΔTB_h is its variation over h. This variation is linked to the risk appetite RA_t and macroeconomic conditions $\mathbf{x}_{h,\Delta}$.

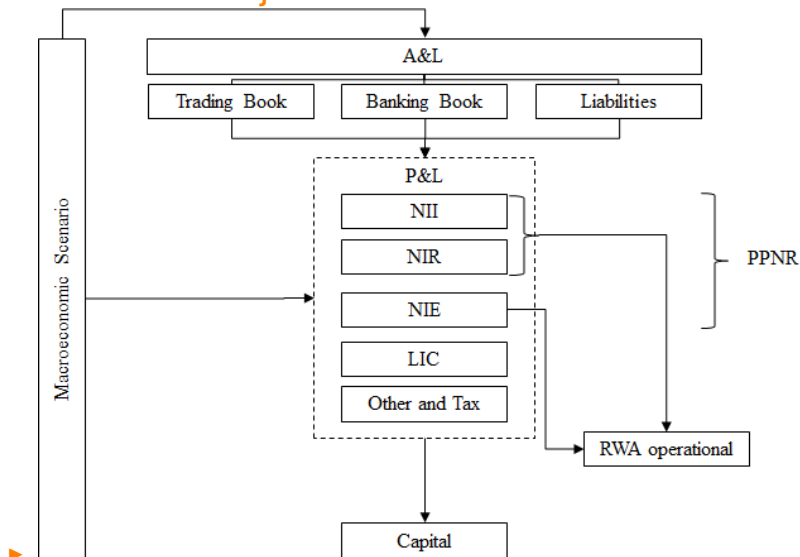▶ A similar approach may be followed for liabilities.

Profit and Loss Projection

Figure: Profit and loss stress testing framework.

Bank Alpha's Stressed Capital Metrics

	t_0	Stress		
		t_1	t_2	t_3
Common equity	4.44	4.33	4.67	5.31
Tier 1	5.44	5.33	5.67	6.31
RWA	76.00	108.00	95.81	88.03
Common equity ratio	5.84 %	4.01 %	4.87 %	6.04 %
Tier 1 ratio	7.16 %	4.93 %	5.92 %	7.17 %

Table: Bank Alpha's capital ratios along the three-year stress ($ billions).

▶ Common equity ratio breaches the minimum 4.50% Basel III threshold in t_1, while the Tier 1 ratio stays below the 6.00% boundary in t_1 and t_2. These results would cause an immediate regulatory intervention followed by a series redemption actions such as capital raising, business restructuring, and so on.

Bank Alpha's Stressed Capital Metrics

▶ Unlike the capital metrics table, the following outlines Bank Alpha's solidity in terms of leverage and LCR.

	t_0	Stress		
		t_1	t_2	t_3
Leverage ratio[2]	5.53 %	5.14 %	5.25 %	5.61 %
LCR[3]	104.00 %	101.00 %	103.00 %	105.00 %
NSFR[4]	88.25 %	94.48 %	94.36 %	93.66 %

Table: Bank Alpha's leverage and liquidity ratios along the three-year stress testing exercise.

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[2]Minimum threshold 3%.

[3]Minimum threshold 100%.

[4]Minimum threshold 100%, consultation period until 2018.

Agenda

- **Stress Testing and Risk Integration**

    - Risk Integration
        - Top-down Approaches
        - Bottom-up Approaches

    - Reverse Stress Testing
        - Objective Functions
        - Internal Features Algorithms

Risk Integration | **Top-Down vs. Bottom-Up**

▶ A major drawbacks of the current regulatory framework is **not to explicitly rely on integrated risk measures**. The recent 2007-2009 crisis highlighted the importance of risk interdependencies.

▶ An interesting literature developed in the last few years around two main approaches:

  ▶ **Top-down** risk integration.

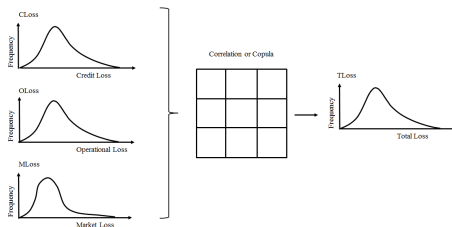 ▶ **Bottom-up** risk integration.

Top-Down |**Basic Integration**

Figure: Top-down risk integration: basic integration.

- A bank total loss may be represented as

$$TLoss_{bi,h} = CLoss_{bi,h} + OLoss_{bi,h} + MLoss_{bi,h}. \qquad (30)$$

- A basic approach may be followed as below

$$TLoss_{VaR,bi,(1-\alpha)} = \sqrt{\sum_{r,s} \rho_{r,s} VaR_{r,(1-\alpha)} VaR_{s,(1-\alpha)}}. \qquad (31)$$
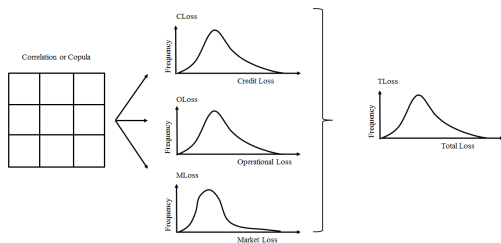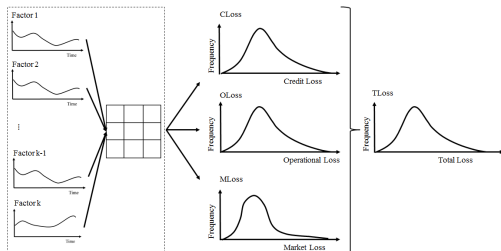
Top-Down |**Top Level**

Figure: Top-down risk integration: top-level approach.

▶ In this case, a copula is used to generate two normal standard variables Y (i.e., credit risk) and Z (i.e., operational risk). Then, the multiplication law of probability is used as follows

$$P(TLoss_{tl,h}) = P(CLoss_{tl,h})P(OLoss_{tl,h}|CLoss_{tl,h}) \tag{32}$$
$$P(MLoss_{tl,h}|CLoss_{tl,h}, OLoss_{tl,h}).$$

Top-Down |**Base Level**

Figure: Top-down risk integration: base-level approach.

- An enhancement of the top-level approach relies on including the analysis of factors influencing copula generations.
- This practice is the step before moving to a fully bottom-up approach.
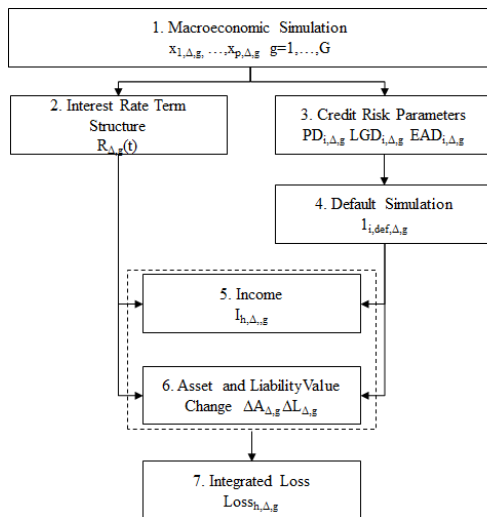
Bottom-up |**Workflow**

Figure: Economic capital risk integration workflow.

Bottom-up | **Economic Capital Objective Function**

▶ Let us suppose that at time t, the value of a bank assets and liabilities are respectively A_t and L_t.

▶ This bank is initially solvent with equity $E_t = A_t - L_t > 0$ (i.e., $A_t > L_t$).

▶ The following equation holds to ensure solvency with a probability $(1 - \alpha)$ over a fixed time horizon h (e.g., one year)

$$P[E_t + \Delta A - \Delta L + I_h > 0] = 1 - \alpha, \qquad (33)$$

where ΔA and ΔL are the asset and liability variations due to macro-economic instantaneous shocks. I_h refers to the income of the period under investigation.

Bottom-up |**Liquidity Objective Function**

▶ The following equation highlights the conditions to ensure solvency with a probability $(1 - \alpha)$ over the time horizon q (e.g., three months)

$$P\big(ALiq_q - LLiq_q + B_q > 0\big) = 1 - \alpha, \tag{34}$$

where $ALiq_q$ represents (asset) liquidity inflows over the period q, $LLiq_q$ stands for (liability) cash outflows and B_q corresponds to the liquidity raised through the market.

▶ The equation above can be re-written as follows

$$P\Big[\textstyle\sum_{t=0}^{q} \Big(\sum_{i=1}^{N} \mathbb{1}_{i,liq,t} A_{i,cf,t} H_{i,t}(\mathbf{x}_t) - \sum_{j=1}^{m} \mathbb{1}_{j,liq,t} L_{j,cf,t}^{nm}(\mathbf{x}_t) + \\ - \sum_{j=m+1}^{M} \mathbb{1}_{j,liq,t} L_{j,cf,t}^{m} + \varrho_t B_t \Big) > 0 \Big] = 1 - \alpha. \tag{35}$$

Reverse Stress | **Intro**

▶ In what follows, a series of alternative objective functions describe how to formally identify conditions leading a bank to fail.

▶ Two broad threatening event categories are studied. On the one hand, internal features are explored as potential sources of bankruptcy. On the other, external economic conditions are investigated while bank operations are considered as given. As a result, a mix of these causes may end in a bank insolvency.

     ▶ **Internal features (Υ)**. The set of asset, liabilities and other internal characteristics representing a bank operational system is denoted with Υ. *Loss* and liquidity mismatching (*LM*), are subsets of Υ. Therefore, Υ, *Loss* and *LM* are jointly considered to have a full picture of a bank.

     ▶ **Macroeconomic scenarios (x)**. A *p*-dimensional macroeconomic vector representing external conditions is the additional component to be studied when assessing potential sources of a bank's insolvency.

Reverse Stress | **Objective Functions - Internal**

► Internal features: economic capital

$$\Upsilon^*(\ell) = \underset{\Upsilon}{argmax} \quad f(\Upsilon|\mathbf{x}, Loss \geq \ell, LM). \tag{36}$$

► Internal features: liquidity mismatching

$$\Upsilon^*(\mathfrak{lm}) = \underset{\Upsilon}{argmax} \quad f(\Upsilon|\mathbf{x}, Loss, LM \leq \mathfrak{lm}), \tag{37}$$

► Internal features: economic capital and liquidity mismatching

$$\Upsilon^*(\ell, \mathfrak{lm}) = \underset{\Upsilon}{argmax} \quad f(\Upsilon|\mathbf{x}, Loss \geq \ell, LM \leq \mathfrak{lm}). \tag{38}$$

Reverse Stress | **Objective Functions - Macro**

▶ Macroeconomic scenarios: economic capital

$$\mathbf{x}^*(\ell) = \underset{\mathbf{x}}{argmax} \quad f(\mathbf{x}|\Upsilon, Loss \geq \ell, LM), \qquad (39)$$

▶ Macroeconomic scenarios: liquidity mismatching

$$(\mathbf{x})^*(\mathfrak{lm}) = \underset{\mathbf{x}}{argmax} \quad f(\mathbf{x}|\Upsilon, Loss, LM \leq \mathfrak{lm}). \qquad (40)$$

▶ Macroeconomic scenarios: economic capital and liquidity mismatching

$$(\mathbf{x})^*(\ell, \mathfrak{lm}) = \underset{\mathbf{x}}{argmax} \quad f(\mathbf{x}|\Upsilon, Loss \geq \ell, LM \leq \mathfrak{lm}). \qquad (41)$$

▶ Internal features and macroeconomic scenarios: economic capital and liquidity mismatching

$$(\Upsilon, \mathbf{x})^*(\ell, \mathfrak{lm}) = \underset{(\Upsilon, \mathbf{x})}{argmax} \quad f(\Upsilon, \mathbf{x}|Loss \geq \ell, LM \leq \mathfrak{lm}). \qquad (42)$$

Reverse Stress | **Trading Book Example**

▶ Let us sample from the trading book portfolio and hypothesize individual or group (i.e., up to p) of debtors' default. The algorithm works as follows.

  ▶ Step 1.

    ▶ Select debtor i. The simplifying assumption of one-debtor, one-financial instrument is followed in order to avoid unnecessary hurdles.

    ▶ If statement. Verify whether debtor the exposure A_i is higher than bank's bankruptcy threshold. If the answer is *yes*, then the loop stops. Otherwise see next step.

  ▶ Step 2.

    ▶ Select a couple of debtors i and j.

    ▶ If statement. Verify whether the outstanding balance invested in debtor i and $j \neq i$, $A_i + A_j$, is higher than bank's bankruptcy threshold. If the answer is *yes*, then the loop stops. Otherwise see next step.

  ▶ Step 3. Continue the process by considering all sets made up by $p \leq n$ debtors.

  ▶ Step 4. All events originating bank's failure are stored.

  ▶ Step 5. All plausible events constitute the output of the reverse stress testing process: $\Upsilon^*_{tradingbook}(\ell)$.
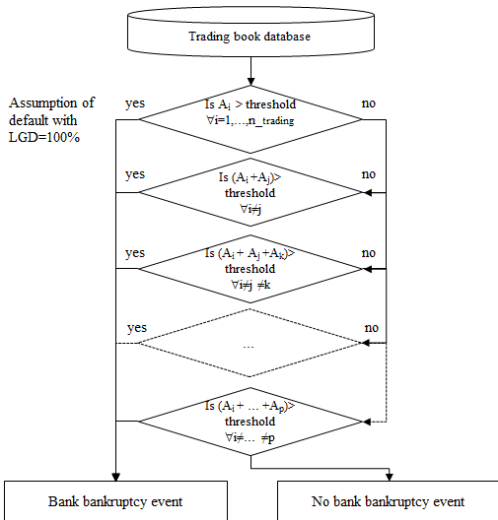
Reverse Stress |Trading Book Example Algorithm 1

Figure: Algorithm 1: trading book individual (joint) default.

Agenda

▶ **Concluding Remarks**

Concluding Remarks

▶ The design of **macroeconomic scenarios** may affect banks stress testing outcomes due to various bank geographic distributions, product types, etc.

▶ Banks capability to face adverse conditions can be assessed through **statistical frameworks**.

▶ Statistical models may fail to highlight potential risks due to their **restrictive assumptions** (e.g., limited risk integration).

▶ A new generation of tools may help prevent new crises by focusing on **micro-structures** and relying on **product-level** information.